HIGH-MIXED-FREQUENCY FORECASTING MODELS – WITH APPLICATIONS TO PHILIPPINE GDP AND INFLATION

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INTRODUCTION

- *Main Topic: technical and practical issues involved in the use of data at mixed and high frequencies (quarterly and monthly and, possibly, weekly and daily) to forecast monthly and quarterly economic activity in a country*
- *Renewed interest in this topic – for timely utilization of high-frequency indicators to update market assessments and forecasts – e.g.,*
  - *Government policy planners*
  - *Investors, industry & business leaders and analysts*

INTRODUCTION (CONT)

Based on Mariano & Ozmucur, Chapter 1 in Peter Pauly (editor) ***Global Economic Modeling***. World Scientific, 2018

Consider high-frequency forecasting models for GDP growth & inflation in the Philippines, focusing on

Multi-Frequency Dynamic Latent Factor Models (MF-DLFM) *and*

Mixed Data Sampling (MIDAS) Regression

Also consider alternative earlier approaches such as

- *AR and VAR benchmark models*
- *Current Quarter Modeling (CQM) with Bridge Equations*

PRACTICAL AND TECHNICAL ISSUES

- Timely and statistically efficient use of breaking "news" for forecast updates --- so use mixed frequency data (quarterly, monthly, weekly, daily, or even intra-day, like tick data in the stock market)
- Prefer data-parsimonious over data-intensive models (without sacrificing forecast accuracy)
- Combination of **mixed-frequency data** and **latent factors** in the dynamic latent factor model introduces additional complexities in the estimation and simulation of the model
- **Data reduction techniques** are needed when dealing with a large number of variables in the data set

MIXED-FREQUENCY DATA SET

- In general, the data set may include quarterly, monthly, weekly, and daily observations
- In this paper,
  - target variables – quarterly
    - GDP and GDP deflator, growth rates
  - indicator variables – monthly (more specific details in subsequent slides)

ALTERNATIVE FORECASTING MODELS (VIS-À-VIS FREQUENCY)

- **Quarterly Models**
  - Use observed quarterly values of target variables
  - Aggregate over the quarter for the monthly indicators,
    - Average over the quarter for a stock variable
    - Sum for a flow variable
    - Calculate growth rates from the aggregated series
- **Monthly Models**
  - Considers all the data series (target or indicator) as generated at the highest frequency (monthly, in our case), but some of them are not observed
  - Variables observed at the low frequency (quarterly) are treated as having periodically missing or unobserved data points, available only at the end month of the quarter

QUARTERLY MODELS

- Benchmark ARMA and VARMA (no indicators used)
  - Y_{tq} ~ ARMA (p.r) or VARMA (p,r)
- Bridge Equations (possibly w/ lags)
  - Y_{tq} ~ [ARMA (p,r), Z_{tq}]
- Bridge – PCA (principal components)
  - Y_{tq} ~ [ARMA(p,r), PC(Z_{tq})]
- CQM – bridge modelling for high-frequency updates of forecasts of GDP and its components

A BIT MORE ON CQM

- Objective: timely forecast of GDP and its components in the national income accounts, typically available quarterly
- Use "bridge" equations, relating GDP components to observabvle quarterly and monthly "indicator" variables.
- Monthly observations are averaged over the quarter, with updates as more monthly observations become available
- To forecast the monthly and quarterly indicators, ARIMA models are used
- If no indicators are available, an ARIMA model would be estimated for the GDP component itself

MONTHLY MODELS (HIGHEST FREQUENCY)

- Mixed-Frequency Vector Autoregressive (MF-VAR)
  - $Y_{tm} \sim [VAR(p), Z_{tm})$
  - Has a state-space model formulation
  - Can use Kalman filtering methods to estimate the model and calculate forecasts at the highest frequency
- Mixed Data Sampling (MIDAS) Regressions
- Mixed Frequency Dynamic Latent Factor Model (MF-DLFM)
- All these models also provide estimates and forecasts of the target variables disaggregated at the high frequency (monthly)

MIDAS

- Initial reference: Ghysels, Santa-Clara, and Valkanov (2004)
- Latest reference: Ghysels & Marcellino (2018). ***Applied Economic Forecasting Using Time Series Methods***. Oxford University Press.

- Early applications – financial; now also used to forecast macroeconomic time series

- More parsimonious parametrization of distributed lag structures to model the relation of GDP to current and lagged indicators at the monthly frequency

$$Y_{tm} \sim DL(Z_{tm}) + \text{error}$$

- Estimation method – Nonlinear Least Squares using actual observed data at mixed frequencies

MIDAS (CONT) - LAG STRUCTURES $\Sigma_K C_K L^K$ (AVAILABLE IN EVIEWS 9.5)

- Step Function (equal weights for months of same quarter, truncated)
- Polynomial Almon Lag
- Exponential Almon

$$c_k = \exp(\theta_1 k + \theta_2 k^2) / \Sigma_k \exp(\theta_1 k + \theta_2 k^2)$$

- Beta Lag $c_k = f(k/K;a,b) / \Sigma_k f(k/K;a,b)$,

$$f(x;a,b) = x^{a-1} (1-x) G(a+b)/[G(a)G(b)]$$

MIDAS (CONT) SOME EXTENSIONS

- Autoregressive MIDAS (*add lag(s) of Y as additional regressor(s)*)
- Unrestricted (but truncated) MIDAS
- Nonlinear MIDAS
- Smooth Transition MIDAS
- Markov-Switching MIDAS
- MIDAS – MF-DLFM or Factor MIDAS (*include latent factors in equation*)

MIXED FREQUENCY DYNAMIC LATENT FACTOR MODEL (MF-DLFM) – ONE COMMON FACTOR

t = time index for the highest frequency

x_t = latent common factor at time t

y_t^i = ith business / economic variables at time t (covers both target and indicator variables

z_t^k = kth exogenous variable at time t

$\tilde{y}_t^i$ = ith observable business / economic indicator at time t

MF-DLFM MODEL

1. Model for latent factor x_t : AR(p) + error

$$\rho(L)\, x_t = \varepsilon_t, \quad \varepsilon_t \sim \text{iid } N(0,1) \ ,$$

$$\rho\,(L) = 1 + \rho\, L + \rho^2 L^2 + \ldots + \rho^p\, L^p$$

2. Model for variables y_t^i (NOT fully observed!)

$$y_t^i = \chi_i + \beta_i\, x_t + \Sigma_\kappa(\delta_{ik}\, z_t^k) + \gamma(L)\, y_t^i + u_t^i$$

$$= [\text{AR}(r)\ ,\ x_t\ ,\ z_t\ ] + \text{error}$$

MF-DLFM
STATE SPACE FORMULATION

Measurement Eq: $y_t = Z_\tau \alpha_\tau + \Gamma \omega_\tau + \varepsilon_\tau \; ; \; \varepsilon_\tau \sim (0, H_\tau)$

State Eq: $\alpha_{\tau+1} = T \alpha_\tau + R\nu_\tau \; ; \; \nu_\tau \sim (0, Q)$

y_t = vector of FULLY observed variables

α_t = vector of state variables

ω_t = vector of predetermined variables such as constant term, trends, exogenous factors, and lagged dependent variables

ε_t = measurement shocks

ν_t = transition shocks

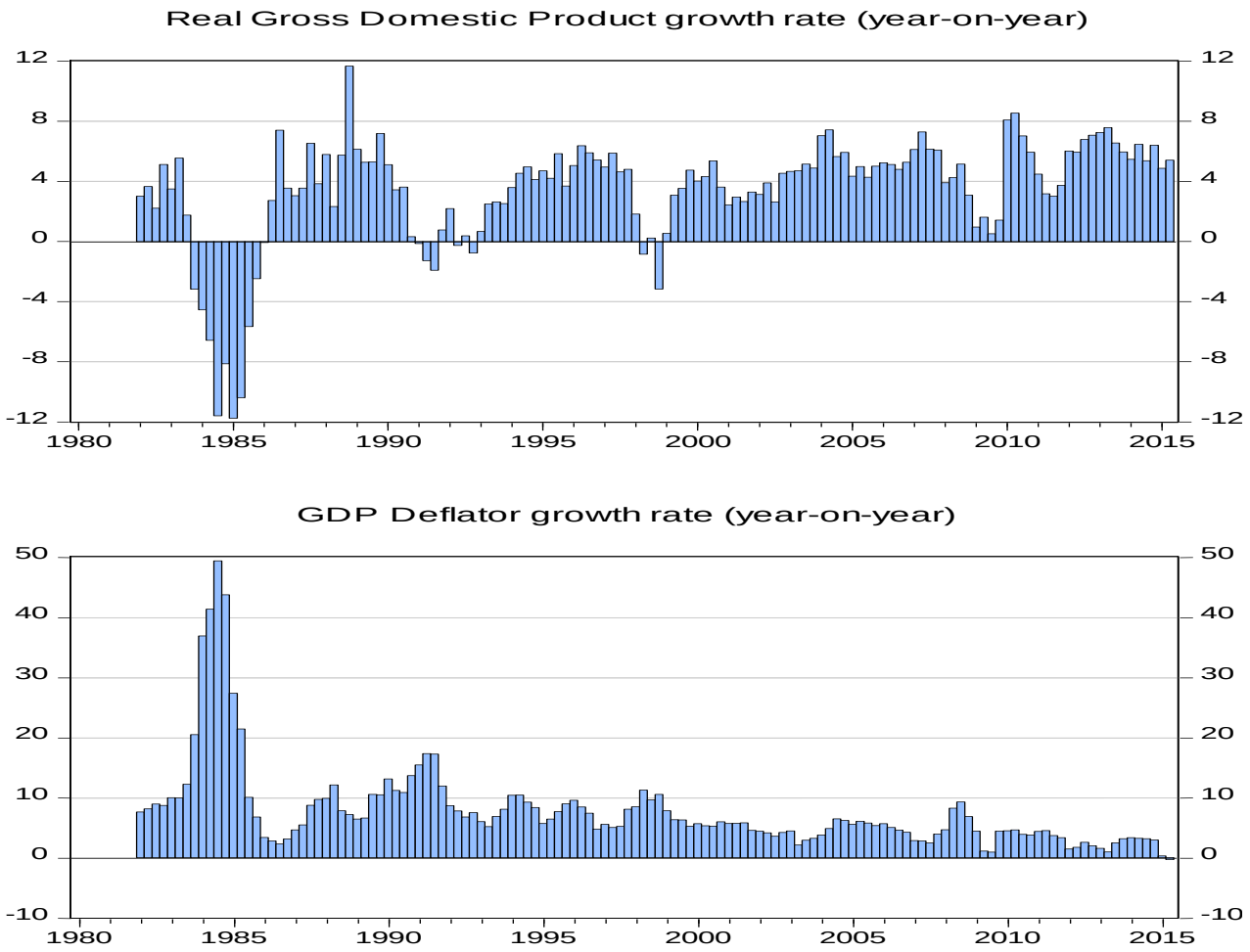
Mariano & Murasawa (JAE 2003, OBES 2010)

Aruoba, Diebold & Scotti (JBES 2009)

EMPIRICAL RESULTS IN THE PAPER - PHILIPPINES

- Two Target Quarterly Variables
  - Real GDP Growth Rate
  - GDP Deflator Growth Rate
- All Indicator Variables are monthly
- Estimation Period
  - 2000.Q1 – 2015.Q4

PHILIPPINE GDP & GDP DEFLATOR GROWTH RATES, 2000-2015



INDICATORS FOR PHILIPPINE REAL GDP GROWTH RATE (Y52)
ALL MONTHLY

- Y-o-y growth rates
  - Industrial production index (Y01)
  - Merchandise imports (Y02)
  - Merchandise exports (Y03)
  - Real government expenditure (Y04)
  - Real money supply (M1) (Y05)
  - World trade volume (Y06)
  - Real stock price index (Y07)
  - Real exchange rate (Y08)
- Y-o-y difference
  - Time deposit rate – savings deposit rate (Y09)
  - Treasury bill rate (91-day) – US treasury bill rate (3-month) (Y10)

INDICATORS FOR PHILIPPINE GDP DEFLATOR GROWTH RATE (Y53)
ALL MONTHLY

- Y-o-y Growth Rates for
  - Consumer price index Y(21)
  - Producer price index (Y22)
  - Wholesale price index, Metro Manila Y(23)
  - Retail price index Y(24)
  - Exchange rate Y(25)
  - Money supply (M1) Y(26)

- Y-o-y differences for
  - Time deposit rate – savings deposit rate (Y09) or Y(29)
  - Tbill rate (91-day) – US Tbill rate (3-month) (Y10) or Y(30)

FORECASTING MODELS ESTIMATED FOR THE PHILIPPINES

- **Quarterly**
  - AR
  - VAR
  - LEI
  - Bridge
  - Bridge – PCA
  - PCA with Two Groups
- **Monthly**
  - MIDAS – polynomial Almon lag
  - MIDAS – PCA
  - MF-DLFM

ESTIMATED BENCHMARK AND LEI

- Estimated quarterly AR models:
  - AR(1) for real GDP growth rate, with $R^2 = 0.49$
  - AR(2) for GDP deflator growth rate, $R^2 = 0.67$

- Estimated quarterly VAR for Y52 and Y53 : VAR(2)

- LEI includes the leading economic indicator index and its lags as additional regressors in the individual quarterly AR models. Estimation results: ARDL(1,1) for real GDP and ARDL(2,0) for the GDP deflator, with $R^2 = 0.58$ and $R^2 = 0.67$, respectively

ESTIMATED BRIDGE & BRIDGE-PCA

- BRIDGE and BRIDGE-PCA equations are estimated separately for the two target variables. These are quarterly data regressions of target variables on the indicators, with monthly indicators converted to quarterly by averaging.

- R^2 values for the estimated Bridge equations
  - 0.74 for real GDP growth
  - 0.89 for GDP deflator growth

ESTIMATED MIDAS

- MIDAS – regressions with Almon polynomial lags are estimated separately for the two target variables. For MIDAS-PCA principal components of the indicators are utilized in the regressions. Results for MIDAS-PCA are not much different from MIDAS; but there are some differences in forecasting performance

- R^2 values: 0.90 for real GDP; 0.95 for the GDP deflator

ESTIMATED MF-DLFM

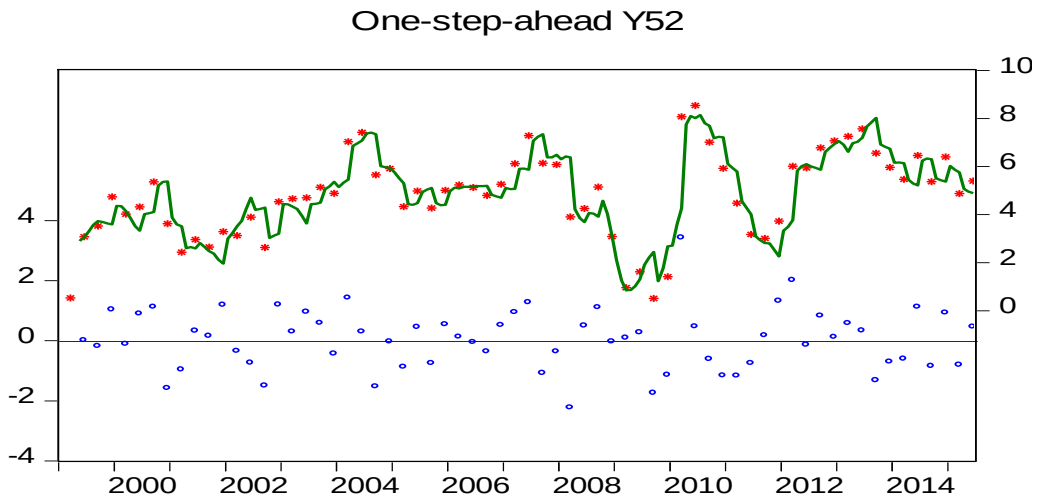
- The two groups of indicators are combined into one and, because of data issues (mostly, timeliness), real government expenditures and the difference between the time deposit rate and the savings rate are excluded.
- A bivariate MF-DLFM model is estimated for the two target variables, with two unobserved common factors (F) and indicator variables W

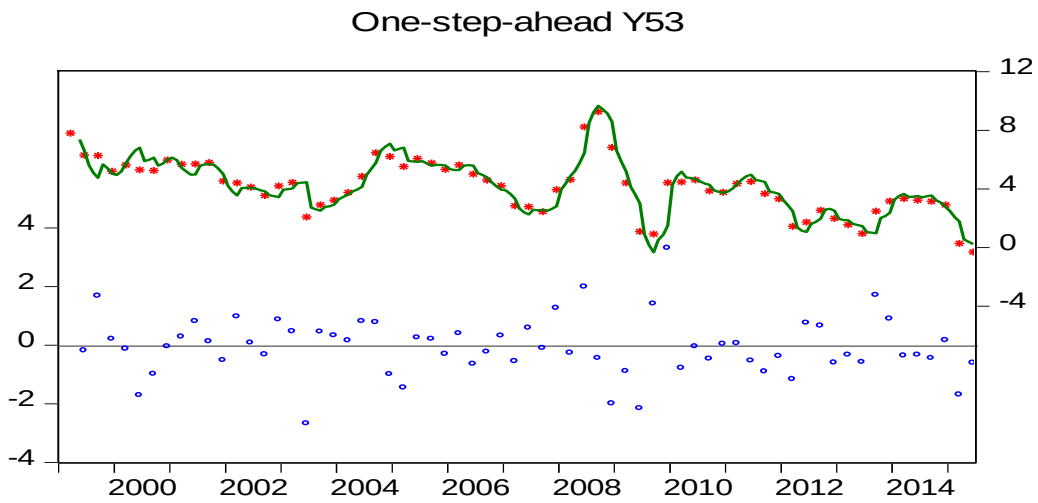
$$Y(t) = B(L)\, F(T) + D(L)\, Y(t-1) + C\, W(t) + v(t)$$

$$F(t) = A(L)\, F(t-1) + u(t)$$

- Maximum Likelihood Estimation is implemented through EVIEWS

MF-DLFM ONE-STEP AHEAD FORECASTS – Y52 & Y53





COMPARISON OF ERROR STATISTICS

- Based on one-period-ahead forecasts, **MF-DLFM has the lowest mean absolute error and root mean square error for GDP growth rate** - .40% for real GDP growth rate, and 0.33% for the nominal GDP growth rate. Corresponding statistics are 0.43%, and 0.49% for MIDAS_PCA, which ranks the second. Principal components, and bridge equations follow these two models. The benchmark AR and VAR models show the biggest errors.

- On the other hand, **MIDAS_PCA has the lowest mean absolute error for the GDP deflator** (0.34%). MF-DLFM, Bridge, and Bridge-PCA MAEs are bunched at 0.47%. AR and VAR models show the biggest errors.

Mean Absolute Errors

One-Period Simulation : 2000.Q2 - 2015.Q4

	Real GDP Growth Rate	GDP Deflator Growth Rate
AR	0.89	0.73
VAR	0.86	0.76
LEI	0.84	0.73
BRIDGE	0.69	0.50
BRIDGE-PCA	0.68	0.44
PCA-2 Groups	0.68	0.48
MIDAS	0.44	0.35
MIDAS-PCA	0.44	**0.34**
MF-DLFM	**0.40**	0.47

Source: Table 1 - Condensed

COMPARISON RESULTS FOR THE PHILIPPINES - DIEBOLD-MARIANO TEST

- Diebold-Mariano statistics were calculated to test the forecast accuracy of MF-DLFM relative to the other models, one at a time. For real GDP growth, test results show statistically significant lower errors for MF-DLFM, except when compared with MIDAS or MIDAS-PCA.
- For the GDP deflator, the MIDAS-PCA average forecast error is the lowest and significantly better than MF-DLFM at 5% critical level.
- MF-DLFM has lower errors relative to Bridge and Bridge-PCA, but not statistically significant at 5%.
- The bivariate tests done here can be extended to a multivariate test comparing MF-DLFM with the alternative methods taken together - see Mariano & Preve (2012)

Diebold-Mariano Statistics for Forecast Accuracy
Alternative Model Versus MF-DLFM
Based on Squared Forecast Errors
Over 2000.Q2 - 2015.Q4

	Real GDP	GDP Deflator
AR	2.45	3.05
VAR	4.02	3.23
LEI	2.74	3.04
BRIDGE	3.90	**1.70**
BRIDGE-PCA	3.92	**0.35**
PCA-2 Groups	3.92	**1.40**
MIDAS	**0.96**	**-1.81**
MIDAS-PCA	**1.16**	**-2.07**

Source: Table 2 - Condensed

COMPARISON RESULTS FOR THE PHILIPPINES – TURNING POINT ANALYSIS

- All models do relatively well, if the prediction is for the level of GDP, real GDP or the GDP deflator. However, not all of them fare well in predicting the turning point in the growth rate of these indicators .

- For the growth rates, MF-DLFM appears to have a bigger edge over the other models in predicting turning points

- DLFM correctly predicts 87% of turning points in real GDP, while MIDAS predicts 74% of them. The ratio is 79% for the bridge equation model, and the PCA model.

- MF-DLFM correctly predicts 89% of downturns, and 85% of upturns. Corresponding ratios for the MIDAS model are 74% and 68%.

TURNING POINT ERRORS

Table 3. Turning Point Errors(2000Q3-2015Q4)

Alternative models	correct total	correct downturn	correct upturn	Pearson c^2	Phi Coefficient
Real Gross Domestic Product Growth (y-o-y)					
AR	0.81	0.83	0.78	23.4	0.61
VAR	0.77	0.70	0.84	18.8	0.55
LEI	0.73	0.73	0.72	12.7	0.45
Bridge Equations	0.87	0.93	0.81	34.8	0.75
PCA_2groups	0.89	0.97	0.81	38.2	0.79
Bridge with PCA	0.89	0.97	0.81	38.2	0.79
MIDAS	0.90	0.93	0.88	40.5	0.81
MIDAS_PCA	0.94	0.93	0.94	47.0	0.87
DLFM	0.90	0.80	0.97	38.0	0.78

OUT-OF-SAMPLE, 2011.Q2 -2015.Q4

		Real GDP growth	GDP Deflator growth	GDP growth
AR	MAE	0.81	1.30	1.00
AR	RMSE	0.97	1.60	1.41
VAR	MAE	0.78	1.35	1.41
VAR	RMSE	0.90	1.63	1.89
LEI	MAE	1.06	1.33	1.07
LEI	RMSE	1.23	1.62	1.44
MIDAS	MAE	0.41	0.35	0.56
MIDAS	RMSE	0.56	0.43	0.72
MIDAS_PCA	MAE	0.52	0.54	0.61
MIDAS_PCA	RMSE	0.73	0.64	0.78
DLFM	MAE	**0.23**	**0.29**	**0.38**
DLFM	RMSE	0.28	0.36	0.47

CONCLUDING REMARKS

- *We have considered alternative models for use of data at mixed frequencies (quarterly and monthly and, possibly, weekly and daily) to forecast monthly and quarterly economic activity in a country*

- *While alternative models are mostly data-intensive, MF-DLFM presents a parsimonious approach which depends on a much smaller data set that needs to be updated regularly. But it also faces additional complications in methodology and calculations as mixed-frequency data are included in the analysis.*

CONCLUDING REMARKS (CONT)

MF-DLFM for real and nominal GDP

MIDAS for inflation

But final verdict is still on hold *– more work needed – e.g.*

- *more elaborate error structures*
- *multiple latent common factors*
- *choice of indicators*
- *multiperiod forecasting (dynamic simulations)*
- *how about other Asian countries?*

THE END