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Commodity Return Predictability: Evidence from Implied Variance, Skewness and their Risk Premia

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Abstract

This paper investigates the role of realized and implied moments and their risk premia (variance and skewness) for commodities' future returns. We estimate these moments from high frequency and commodity futures option data that results in forward-looking measures. Risk premia are computed as the difference between implied and realized moments. We highlight, from a cross-sectional and time-series perspective, the strong positive relationship between commodity returns and implied skewness. Moreover, we emphasize the high performance of skewness risk premium. Additionally, we show that portfolios built by sorting skewness risk premium and implied skewness exhibit the best risk-return tradeoff. Most of our results are robust to other factors such as the momentum and roll yield.

JEL codes: G13; G17.

Keywords: Commodity Forecast; Implied Volatility; Implied Skewness; Risk Premium.

I. Introduction

Several existing studies support the view that investors are not only concerned about volatility but skewness as well (Kumar, 2007, 2009; Barberis and Huang, 2008; Mitton and Vorkink, 2007). As such, given their preferences for low volatility and positive skewness, investors might require a premium to hold high volatile and negatively skewed portfolios. Moreover, while recent research highlights the predictive power of skewness for equity returns, the evidence is limited in commodity markets. Our paper contributes to this ongoing debate, shedding light on the role of these variance and skewness moments and their risk premia in commodity markets. Their understanding is of relevance to market participants, policymakers, and academics, as well.

In this study, we explore the predictive ability of commodities' realized (physical) and implied (risk-neutral) moments and their risk premia over their returns. Specifically, we investigate whether the variance, skewness, and their risk premia can forecast commodity future returns from both a time-series and a cross-section perspective. We compute the realized variance and skewness using high-frequency data for each of the eight most liquid commodity futures contracts, namely, agricultural (corn, soybeans, wheat), metal (copper, silver, gold), and energy (oil, natural gas) commodities. As Amaya et al. (2015) point out: "*... skewness (and kurtosis) measures computed from high-frequency data are likely to contain different information from those computed from daily data or from options*". Using the model-free approach of Bakshi et al. (2003), we then estimate the implied variance and skewness moments and their risk premia, defined as the difference between risk-neutral and physical moments.

By taking into account the different information stemming from option and high-frequency data, we uncover several interesting results, especially about skewness.[1] First, we show that from both a time-series and cross-section perspective, generally, there is a strong positive and significant relation between implied skewness and future commodity returns. Regarding the skewness risk premium (SRP), defined as the difference between implied and realized skewness, while from a time-series perspective, there is only a statistically significant positive relationship for the agricultural commodities soybeans and wheat, from a cross-section perspective, its portfolio exhibits the highest positive and significant performance.

[1] We also use the daily returns to compute the realized variance and skewness and estimate their risk premia. In general, in line with our empirical evidence, results suggest the existence of a limited predictive power of the risk premia. The results are available on request.

Notably, a trading strategy with futures contracts entering long on the commodities with the highest implied skewness and short on the commodities with the lowest implied skewness yields average annualized returns of 17.21% and annual volatility of 28%. The results are better when forming portfolios using the SRP, with average annualized returns of 18.37% and annual volatility of 25.9%. These findings empirically support the existence of arbitrage opportunities between futures and options in commodity markets. We also find that although the strategy on variance risk premium (VRP) is weaker than that on implied skewness and SRP, it nevertheless provides a reasonable risk-return tradeoff. Second, by constructing an efficient frontier with commodities, stocks, and bonds, we highlight the importance of implied skewness and SRP portfolios from the U.S. investor's viewpoint. We show that these portfolios display the best risk-return tradeoff and relevant weights in the efficient frontier portfolios with equivalent risk as that of bond, equity, and commodity markets.

The intuition behind our finding of a positive relationship between implied skewness and future commodity returns is based on the informed trading and hedging views in equity markets documented by Stilger et al. (2017) and Xing et al. (2010). Particularly, if there are arbitrage opportunities and information differences between spot and derivative markets, then the positive implied skewness predicts positive future returns. Moreover, this relationship also relates to Kang and Pan's (2015) model, where commodity producers hedge their risk by trading in both futures and options markets, and speculators provide liquidity. Hence, in our study, we posit that this producers' simultaneous demand for derivative contracts leads to a positive predictive relationship between implied skewness and commodity returns.

Contrary to previous views, the skewness preference theory (Bali and Murray, 2013; Conrad et al., 2013) predicts a negative relation between implied skewness and future returns, which is inconsistent with our empirical findings for commodities. According to this theory, the intuition is that, in the absence of arbitrage rules between stock and option markets, the same information should be reflected in both markets. Therefore, the positive implied skewness would predict negative expected stock returns and vice-versa. However, it is unclear whether this theory can apply to commodity markets. While a negative skewness in stock markets indicates that investors might want to hedge against an equity market decrease, for commodity markets, it has different implications for either producer (or investor) and consumer. Commodity producers might want to hedge against price drops, whereas consumers would like to hedge against price upswings. Given that skewness preference in commodity markets might be either positive or negative, we are unable to draw conclusions about future returns.

Regarding the intuition of the positive relation between the SRP and future returns, we rely on the asymmetric information setup where specific agents have superior information about asset outcomes. For instance, let us assume that option prices reflect the information held by market makers (but not by other agents) about future outcomes. If market makers fear a price drop, they would charge a higher margin against physical probabilities for the out-of-the-money puts (left tail) compared to out-of-the-money calls (right tail). In this way, we would have a negative SRP. When the negative anticipated outcome is revealed, prices will drop. The other way around, if market makers fear a price spike, they would charge a higher margin for out-of-the-money calls, leading to a positive SRP. When the positive outcome is revealed, prices will go up. Therefore, we would have a positive relationship between SRP and future returns. We might have a similar setup if hedgers, but not market makers, have superior information. However, in this case, the demand pressure will play a role together with asymmetric information.

Our paper contributes to two strands of the literature. First, we add to the literature on return predictability that considers realized and implied moments. Regarding stock markets, Amaya et al. (2015) and Choi and Lee (2015) find support for a negative theoretical relation between physical skewness and expected returns. Instead, the literature on stock predictability of implied skewness reports contradictory results. While Conrad et al. (2013) and Bali et al. (2011) find a negative relation between implied skewness and future returns, other studies document a positive relation (Rehman and Vilkov, 2012; Cremers and Weinbaum, 2010; Xing et al., 2010). For commodity markets, as far as we know, only one study examines the role of realized skewness in predicting future returns (Fernandez-Perez et al., 2018), and two other studies focus on the implied variance (Chatrath et al., 2016; Gao, 2017). By studying whether implied variance and skewness could forecast commodity's returns, we extend the literature on their mixed predictability findings for stock markets to a different asset class, where its evidence is either narrow or absent. In addition, note that our study uses high-frequency data to construct realized moments, especially realized skewness, whereas Fernandez-Perez et al. (2018) rely on daily data. Therefore, our study provides new insights regarding the role of both realized and implied moments for commodity returns' predictability. By focusing on the implied volatility's role in predicting commodities' returns we add to the study of Hollstein et al. (2020) who investigate the spillovers in principal components of the implied volatility term structures of commodity markets and their drivers. We also complement Hollstein et al. (2021) who examine whether a large set of equity return anomalies including skewness are priced in the cross-section of commodity futures returns. In particular, our study i) uses high-frequency data to compute the realized skewness (rather than a low-frequency as in the above study), ii) considers the skewness risk premium, and iii) explores the commodity return predictability from both time-series and cross-sectional perspectives

Moreover, while studies on commodity return predictability generally concentrate on classical commodity theories such as the theory of storage, our study puts forward investors' preferences for volatility and especially skewness as possible determinants of future commodity return. In particular, it contributes to this literature by uncovering the important role of skewness and skewness risk premium beyond the fundamental concepts of backwardation and contango. Furthermore, we show that our results are not driven by various commodity and equity-specific factors such as the roll yield, momentum, Fama and French (1993) factors, and the dollar index.

Second, we contribute to the literature on return predictability that focuses on variance and skewness risk premia. Its empirical evidence points out a positive relation between VRP and future returns in stock markets, namely, higher (lower) variance risk premium predicts higher (lower) future returns (Bollerslev et al., 2014; Bollerslev et al., 2009). Instead, for currency (Ornelas, 2019; Della Corte et al., 2016; Londono and Zhou, 2017) and commodity markets, especially the oil market (Chevallier and Sevi, 2014; Triantafyllou et al., 2015; Ornelas and Mauad, 2017), the direction of this relationship is controversial. Chevallier and Sevi (2014) document a negative relation between the VRP and crude oil's returns. Similarly, Triantafyllou et al. (2015) confirm the negative relation for agricultural commodities (wheat, corn, and soybeans). In contrast, Ornelas and Mauad (2017) find a positive relationship for both oil and gold commodities. As a proxy of implied volatility, these studies use the Chicago Board Options Exchange (CBOE) volatility index relying on various exchange-traded securities. Our paper contributes to this literature by investigating the predictability of other implied moments using options on commodity futures contracts.

Still, on the commodity literature, Ruf (2012) finds evidence that net option exposures from long speculators can forecast commodities' implied skewness and skewness risk premium (i.e., the difference between realized and implied skewness). In addition, the author proposes a profitable trading strategy that explores this predictability by building portfolios using options. For instance, a portfolio would go long (short) in implied skewness in commodities with above-normal (below-normal) levels of speculative long exposure in options. Although this trading strategy is implemented with options, instead of futures contracts like in our case, the idea behind it is similar, i.e., to obtain exposure to the risk premium coming from a set of commodities and earn abnormal returns. Prokopczuk et al. (2017), instead, focus more on exploring the comovement among the commodity variance swap payoffs (i.e., realized and expected variance swap payoffs) and between them and commodity, equity, and bond returns. Similarly, Trolle and Schwartz (2010) investigate the time variation of energy variance risk premia and the factors driving it by relying on variance swaps on futures contracts. Our study complements the above studies by i) using high-frequency data to calculate the realized variance and skewness as the floating legs of the skewness and variance swaps, whereas the previous studies rely on low-frequency data for their computation, and ii) analyzing the commodity return predictability rather than exploring the comovement among the commodity variance swap payoffs.

Our study further complements the literature on the role of skewness risk premium in stock markets (Harris and Qiao, 2018; Bali et al., 2019; Lehnert et al., 2014). From a cross-section perspective, these studies show a positive relation between SRP and equity returns.[2] To the best of our knowledge, we add to this literature by being the first paper to investigate and provide strong evidence about the predictability power of SRP and implied skewness over commodity returns.

The remainder of the paper is structured as follows: Section II describes the data. Section III and IV show the forecasting results from a time-series and cross-section perspective. Section V presents the robustness tests. Section VI concludes.

II. Data

Analysis of commodity return predictability relies on option and high-frequency nearby futures data from Thomson Reuters Tick History and daily nearby futures data from Bloomberg. Our sample consists of eight commodities that cover three main sectors, namely, agricultural (corn, soybeans, and wheat), metal (copper, silver, and gold), and energy (WTI oil and natural gas), from January 2008 to December 2016.[3] We calculate daily returns and the one-month, two-month, and three-month returns of the continuous first nearby futures contracts for each commodity. We compute the realized variance and skewness by using the sums of five-minute returns, including

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[2] It is worth noting that all these studies define differently the skewness risk premium: as the realized skewness minus implied skewness or physical minus risk-neutral skewness. As their definition is inverse of ours, the sign of their relationship is also inverse.

[3] We choose the most liquid commodities options from their representative sectors and cover a sample period where both realized and implied moments are reliable to balance the cross-section and time dimensions. We acknowledge that it would be interesting to extend this research to other commodity markets and for a longer sample period. However, we note that the liquidity of those markets may be problematic in the calculation of especially risk-neutral measures based on options data. For instance, Ruf (2012) acknowledges that commodity futures' data from CRB exhibits some issues, which may be subject to criticism. Specifically, the author emphasizes that the lack of bid or ask quotes creates difficulty in judging how noisy individual quotes are and how much of this noise trickles down into the non-parametric skewness measures. Using the same data provider, Prokopczuk et al. (2017) observe many variations in trading activity across commodity sectors and recognize that the variance swaps of individual commodities might be noisy.

the overnight returns as in Amaya et al. (2015). Following the well-known model-free approach of Bakshi et al. (2003), we then estimate the implied moments, namely, implied variance and skewness, using one-month options on futures contracts. Finally, we define risk premia as the difference between implied and realized moments (Bollerslev et al., 2009). Appendix A provides more details of the high-frequency and option data.

Table 1 provides summary statistics for the realized (Panel A) and implied (Panel B) moments and their risk premia (Panel C). We present the mean and standard deviation and the 25th and 75th quantiles for each agricultural, energy, and metal commodity sector. The implied moments (Panel B) are in general higher than the realized moments (Panel A), and as such, the risk premia statistics (Panel C) are positive on average. These statistics align with the literature that finds higher average implied variance values (see, e.g., Trolle and Schwartz, 2010; Prokopczuk et al., 2017; Hollstein et al., 2020).[4]

INSERT TABLE 1 HERE

III. Time-Series Predictability

This section shows the univariate outcomes on the predictive ability of commodities' realized and implied moments and risk premia (variance and skewness) for their returns. We assess this predictability for one-month returns. We then discuss whether their predictability holds for one-month returns when accounting for the impact of well-known commodity factors, i.e., roll yield and dollar index.

[4] For instance, using a different sample period and method for the implied volatility's estimation (i.e., January 1996 to December 2015), Hollstein et al. (2020) report an average annual implied volatility of 29%, 25%, 31%, 32%, 18%, 36%, and 50% for the corn, soybeans, copper, silver, gold, oil, and natural gas, whereas, in our study, we find around 33%, 26%, 42%, 32%, 20%, 37%, and 40%. In addition, Simon (2002) finds an average mean of the agricultural implied volatility that is usually lower than that of ours (e.g., the annual implied volatility of corn, soybeans, and wheat is around 22.34%, 22.32%, and 22.25%). Thus, the implied variances would also provide consistent estimates. The small differences for some commodities may be related to either the sample period, method for implied volatility estimation, providers of options data (e.g., Commodity Research Bureau (CRB), TRTH), or a combination of these factors. Indeed, even when using a similar sample period from around 1990 to 2010 and the same data provider but a different approach for the implied volatility's estimation, Ruf (2012) and Prokopczuk et al. (2017) document the existence of heterogeneity across commodities in term of the magnitude of implied volatilities and variance swap payoffs, respectively. For example, although both the implied variance swap payoffs and implied volatility are higher for natural gas than wheat, there is a difference when comparing their magnitude (i.e., the implied variance in Prokopczuk et al. (2017) is 28.336% and 7.82% whereas the implied volatility in Ruf (2012) is 47.3% and 26.5%). Given the above and especially, that both papers' sample period ends around the beginning of our sample, we consider that it is expected and reasonable to observe some differences in the statistics.

III.A Univariate Results

From a univariate perspective, this section addresses the predictive ability of variance and skewness for the commodity returns over one-month. Specifically, we investigate their role by distinguishing between realized and implied moments and considering their risk premia.

The regression specification for the realized, implied, and risk premium moment's predictability is given by:

$$Ret_{i,t,h} = \alpha + \beta_1 * M_{i,t-1} + \beta_2 * Ret_{i,t-h,h} + \varepsilon_{i,t} \quad (1)$$

where $Ret_{i,t,h}$ is commodity *i* return starting at business day *t* for a holding period *h* – one-month for baseline results and two- and three-month for the additional results from the Appendix. $M_{i,t-1}$ is each of the realized, implied, and risk premium moments, i.e., variance or skewness, of each commodity *i* for a period of one-month. The realized moments' window uses a five-minute sampling frequency that starts one month before (21 business days) and ends at business day *t-1*. As for implied moments, we use option prices traded at time *t-1* with expiration in one month.

Regressions cover the period from 2008 to 2016 on a rolling daily basis for our eight most liquid commodities, namely, agricultural (corn, soybeans, wheat), metal (copper, silver, gold), and energy (oil, natural gas) sectors. Given our strongly overlapping sample, we use the Hansen-Hodrick t-statistics. Table 2 presents Equation (1) results in Panels A and B, for the realized and implied moments, and in Panel B, for their risk premia.

Examining Panel A, we observe no significant relation between realized variance and skewness and future one-month returns. It is worth mentioning, however, that realized moments of copper and oil show certain predictive ability when considering two-month and three-month returns (see Table A.3 from the Appendix).

Panel B highlights the remarkable good predictability of implied skewness for commodity returns. In particular, note that six out of the eight commodities have positively statistically significant coefficients. These commodities include corn, soybeans, wheat, gold, oil, and natural gas. The results are in line with the presence of informed derivatives trading and arbitrage opportunities between commodity derivative markets, already proven and documented in equity spot and option markets (Stilger et al., 2017, Xing et al., 2010, Rehman and Vilkov, 2012; Cremers and Weinbaum, 2010). Additionally, implied skewness' predictability of corn, wheat, and natural gas carries on being significant for their two-month and three-month returns as well (see Table A.4 from the

Appendix). Implied variances are good predictors only for metals' longer horizons (see Table A.4 from the Appendix).

Panel C shows that, generally, risk premium coefficients are positive. However, most of them are not statistically significant. These findings indicate that the predictive power of implied and realized moments taken together, namely, of the risk premium moments, is less strong than when separately considering the implied moments, as shown in Panel B. For instance, among agricultural VRP coefficients, only those of corn are statistically significant, although negative. These outcomes are contrary to Triantafyllou et al. (2015), who document a negative relation between the VRP of corn, soybeans, and wheat and their two-month returns. We point out that the authors define VRP as opposite to our study, namely, as the difference between realized and implied variance. As such, according to our definition, their negative relation translates into a positive relation in our analyses. The different conclusions might be due to different sample periods, e.g., their samples end in December 2011, and different computations of VRP. That is, the authors focus on the two-month VRP and estimate the two-month realized variance using daily prices. In contrast, our paper considers the one-month VRP and high-frequency data for computation of the realized variance.

Regarding VRP coefficients of metal commodities, we observe that their coefficients are positive and only statistically significant for copper. Gold's positive coefficients align with Ornelas and Mauad (2017), who document a positive relationship between the CBOE Gold Volatility Index (GVX) and its returns. However, their relationship is statistically significant, whereas ours is not. Our silver coefficients are also positive and nearly statistically significant. Coefficients of oil are negative but statistically insignificant. Their signs are in line with those in Chevallier and Sevi (2014), who use the CBOE Crude Oil Volatility Index (OVX).

We further observe that, in general, there is a positive but not statistically significant relation between commodities' skewness risk premium and their future returns. Exceptions are the positive and statistically significant return coefficients of agricultural commodities (wheat and soybeans).

INSERT TABLE 2 HERE

A possible explanation for our findings might be related to the computation of skewness risk premium, namely, the sampling frequency of realized skewness. Several studies that analyze the skewness risk premium use daily U.S. equity returns to compute realized skewness (Harris and Qiao, 2018; Bali et al., 2019; Lehnert et al., 2014). Nevertheless, as Amaya et al. (2015) has stated: "*We conclude from these general results for the third and fourth realized moments that we can* *expect very different estimates of skewness and kurtosis depending on the frequency of data used to estimate these moments. Skewness estimates from moving windows of daily or weekly data are likely to have different averages than skewness measures constructed from intraday data*". In this way, it is most likely that skewness risk premium could provide different results by using different periodicities in estimating the realized skewness. This conclusion contrasts with the second moment, i.e., variance, where a higher frequency of returns leads to better and more efficient estimates. In our study, we compute the skewness risk premium using high frequency (five-minute) returns. Thus, using another sampling frequency might provide different results. As addressing the best sampling frequency for estimation of skewness risk premium is not the scope of our paper, we let this debate for future research.

III.B Multivariate Results

In this section, we add several control variables to the previous time-series analysis. Specifically, we use two known commodity predictors, such as the roll yield and dollar index. Roll yield is the yield obtained from the rolling of a short-term futures contract to a long-term futures contract. Hence, the yield an investor receives when its futures contract position converges to the spot price. Note that the roll yield is inversely correlated with the slope of the term structure of futures contracts. Following Arnott et al. (2014), we use the one-year roll yield that relies on the first nearby contract and its next-year counterpart, namely:

$$RY_{i,t} = LN\left(\frac{C_0}{C_1}\right) \tag{2}$$

where $RY_{i,t}$ is the roll yield of commodity *i* at time t, C_0 is the price of the first nearby (front-end) contract of commodity *i*, and C_1 is the price of its next-year counterpart. It is worth mentioning that many papers use the slope between the nearest two contracts in the curve to calculate the roll yield (e.g., Fernandez-Perez et al., 2018). However, Arnott et al. (2014) point out that this method has several drawbacks, including seasonality and homogeneity problems across commodities' available contracts. Further, it is well-known that commodity prices move in the opposite direction from the U.S. dollar. For this reason, the other control variable is the dollar index measuring the value of the U.S. dollar against a basket of currencies. The higher is this index, the higher is the U.S. dollar value against the basket. We use the one-month return of the dollar index.

We start this analysis by using the realized moment's predictability with the following specification:

$$Ret_{i,t,h} = \alpha + \beta_1 * RM_{i,t-1} + \beta_2 * RY_{i,t-1} + \beta_3 * DXY_{i,t-1} + \beta_4 * Ret_{i,t-h,h} + \varepsilon_{i,t} \quad (3)$$

where $RM_{i,t-1}$ is the realized moment of commodity *i* at time *t-1*, $RY_{i,t-1}$ is the roll yield of commodity *i* at time *t-1*, and $DXY_{i,t-1}$ is the dollar index[5] return at time *t-1*. Table 3 shows the results considering the one-month returns.

Panel A reveals similar results to the univariate case, namely, realized variance has no predictability for one-month returns, except those of natural gas. The roll yield coefficients are generally negative and statistically significant, except for those positive precious metals. In line with commodity literature (e.g., Chen et al., 2014), the dollar index coefficients are mainly negative but present limited statistical significance. Examining Panel B of Table 3, the predictability of realized skewness when including commodity factors, we find akin results to Panel A.

INSERT TABLE 3 HERE

Table 4 presents the implied moment's predictability with the following specification:

$$Ret_{i,t,h} = \alpha + \beta_1 * IM_{i,t-1} + \beta_2 * RY_{i,t-1} + \beta_3 * DXY_{i,t-1} + \beta_4 * Ret_{i,t-h,h} + \varepsilon_{i,t} \quad (4)$$

where $IM_{i,t-1}$ is the implied moment of commodity *i* at time *t-1*, $RY_{i,t-1}$ is the roll yield of commodity *i* at time *t-1*, $DXY_{i,t-1}$ is the dollar index return at time *t-1*.

Panel A of Table 4 shows the predictability of implied variance for commodity returns. Copper's coefficient is again positive and significant, as in the univariate approach. Corn's coefficient is now statistically significant yet negative. Panel B highlights the good predictive power of implied skewness for commodity returns in the presence of additional commodity factors. Note the statistical significance for most of our coefficients, such as corn, wheat, gold, and natural gas. Exceptions are the coefficients of soybeans and oil, which are no longer significant. Surprisingly, copper's coefficient is the only negative and statistically significant coefficient.

INSERT TABLE 4 HERE

We next analyze the predictive role of moment risk premia. The specification is the following:

$$Ret_{i,t,h} = \alpha + \beta_1 * MRP_{i,t-1} + \beta_2 * RY_{i,t-1} + \beta_3 * DXY_{i,t-1} + \beta_4 * Ret_{i,t-h,h} + \varepsilon_{i,t} \quad (5)$$

[5] The dollar index is a measure of the U.S. dollar's value relative to a basket of currencies from its most significant trading partners. A positive (negative) return of the dollar index indicates that the U.S. dollar is gaining (loosing) value.

where $MRP_{i,t-1}$ is the moment risk premium of commodity *i* at time *t-1*, $RY_{i,t-1}$ is the roll yield of commodity *i* at time *t-1*, $DXY_{i,t-1}$ is the dollar index return at time *t-1*. Table 5 presents these results.

In Panel A, we find that VRP coefficients of corn and copper are still significant even after controlling for commodity factors. Instead, looking at Panel B, none of the skewness risk premium's coefficients is significant.

INSERT TABLE 5 HERE

Overall, this section's findings emphasize that, from a time-series perspective, implied skewness is our best variable to forecast commodity returns. The roll yield also has a good forecast ability.

IV. Cross-Section Predictability

In this section, we begin by exploring the performance of several trading strategies relying on realized, implied, and risk premium moments, i.e., variance and skewness. We then compare their performance with other known strategies, such as equal-weighted portfolio, roll yield, and momentum.

The portfolios are built using six types of measures, namely, realized variance, realized skewness, implied variance, implied skewness, variance risk premium, and skewness risk premium. Specifically, we build long-short (cash neutral) portfolios by taking a long position on top 25% measures (realized, implied, and risk premium moments) and a short position on bottom 25% measures. As such, our portfolio contains two long and two short commodity futures contracts with equal weights. We form these portfolios every day and hold them for one month (21 overlapping business days). Thus, each portfolio weighs 1/21. The daily rebalancing gives robustness to our results against outliers and assures results are not coming from a particular day of the month[6].

To benchmark these strategies against other well-known commodity strategies, we build three additional benchmark portfolios. The first benchmark portfolio is the simple, equally-weighted average of the eight commodities in our sample, and thus, it is a long-only portfolio. The second benchmark portfolio relies on the past one-year performance of our commodities, i.e., the one-year momentum, buying past one-year winners, and selling past one-year losers. Finally, the last benchmark portfolio uses the one-year roll yield (inverse of slope's futures contract term structure),

[6] Besides being more sensitive to outliers, rebalancing the portfolio once a month (instead of daily) would possibly force the portfolio manager to go deeper in the order book, with higher transaction costs, especially for large amounts. Daily rebalancing has the advantage of spreading the volume of trading during the month.

buying the highest one-year roll yield and selling the lowest one-year roll yield. The momentum and roll yield portfolios are long-short portfolios with the same characteristics as the main portfolios in terms of, for instance, the formation rules, and rebalancing, except for their different sorting criteria. Table 6 presents the portfolio return statistics.

When exploring Table 6, we observe that implied skewness and SRP portfolios exhibit the highest positive and significant performance with their Sharpe ratio over 0.6 and 0.7, respectively. Specifically, these implied skewness and SRP portfolios yield an average yearly return of 17.21% and 18.37%, with an average yearly volatility of 28% and 25.9%, respectively. Transactions costs are unlikely to significantly reduce these returns.[7]

We further document an insignificant performance of other primary and benchmark strategies. Although their performance is insignificant, we next discuss whether these strategies conform to commodities' literature. For instance, the negative return of our realized skewness portfolio is consistent with that in Fernandez-Perez et al. (2018).[8] However, our results are statistically insignificant, whereas previous authors find statistically significant results. Their high number of commodities, different sample periods, as well as the use of daily data for estimation of realized skewness might explain this difference in the statistical significance of the results. The negative mean return of the momentum portfolio is consistent with several studies (Daniel and Moskowitz, 2016; Bianchi et al., 2015; Moskowitz et al., 2012). For instance, Bianchi et al. (2015) find a consistent and strong reversal pattern of commodities' momentum profits from 12 to 30 months. Daniel and Moskowitz (2016) show that equity momentum strategies experience negative returns, especially in panic states such as financial crises or market crashes when volatility is high.

Further, the mean return of the roll yield portfolio is negative. Although this finding is not in line with existing literature on commodity markets, the negative return could be because most commodities have been in backwardation since mid-2012 (Arnott et al., 2014). Indeed, dividing our sample into two sub-periods, namely, from January 2008 to June 2012 and from July 2012 to December 2017, we find a positive and negative mean return of the roll yield portfolio during the former and latter sub-period. That is, while during the first sub-period, the mean portfolio return is

[7] Considering a flat broker fee of USD 2 per contract, it would cost half a basis point for a full portfolio rebalancing. Thus, for rebalancing, we would have a cost of five to six basis points every month considering recent prices. The cost of rolling futures would be lower than rebalancing, but with a similar magnitude.

[8] Note that authors build the realized skewness portfolio by buying commodities with low skewness and selling those with high skewness. Thus, the positive return of their portfolio means a negative return for our portfolio.

3.10% with a *t*-statistic of 0.16, during the second sub-period, it is -23.79% with a *t*-statistic of -2.04.

INSERT TABLE 6 HERE

Figure 1 displays the performance of six commodity strategies using the realized, implied, and risk premium moments, namely, variance and skewness. It highlights, starting in 2012, the increasing performance of investment strategies using the implied skewness and SRP. Moreover, although until 2012 the VRP strategy performs better, towards the beginning of 2015, implied skewness and SRP strategies surpass it. Figure 1 also emphasizes that, generally, the worst performances are due to strategies relying on variance (realized and implied) and realized skewness. The exception was during the Global Financial Crisis (GFC) when the performance of realized skewness strategy is like that of VRP.

INSERT FIGURE 1 HERE

V. Robustness

In this section, we examine the portfolio return statistics using a double sorting strategy and considering various holding periods. We then present the portfolio return statistics using single and double sort strategies depending on the innovations for each of the realized and implied moments and risk premia of our commodities. Additionally, we examine their correlation matrix. Finally, we estimate factor regressions using both original variables and their returns; we present the frequency of commodities entering portfolios and briefly mention other robustness tests done.

Table 7 presents the portfolio return statistics for double sorting strategies with a one-month holding period. Given the significant performance of implied skewness and skewness risk premium in Section IV, we construct double sort portfolios considering them together and on each of them and variance risk premium. All three strategies earn positive annual returns with a significant and relatively higher performance than strategies in Section IV. Specifically, the skewness risk premium and both variance risk premium and implied skewness double sorting portfolios have the best performance with a Sharpe ratio of 0.75 and 0.65, respectively. Besides, we build double sorting portfolios considering the roll yield and each of the implied and risk premium moments. Their portfolio return statistics are insignificant, and thus we do not report them.

INSERT TABLE 7 HERE

Table 8 shows the portfolios return statistics using single and double sorting strategies for a holding period of two weeks (Panel A) and two months (Panel B), respectively. Panel A's results emphasize the significant high performance of the single and double sorting portfolios, as shown in Section IV and previous Table 7. Also, notice that for a two-week holding period, the variance risk premium has a significant performance with a Sharpe ratio of 0.73. Instead, the performance of a double-sorted portfolio on skewness risk premium and implied skewness is not statistically significant. Considering a longer holding period, namely, two-month, Panel B shows that no other trading strategies are significant except momentum's reversal portfolio.

INSERT TABLE 8 HERE

In Section IV, we document a high correlation among commodity portfolio returns. Thus, following Chang et al. (2013), we also use the commodity innovations to estimate all our analyses as an alternative robustness test. Particularly, to obtain these innovations, we fit an autoregressive moving average, ARMA (1, 1), model for each of the realized and implied moments and risk premia of commodities, as well as for their roll yield and momentum. Then, we use these ARMA residuals as time-series innovations. Our univariate, multivariate, and cross-sectional results using innovations are consistent with the realized, implied, and risk premium moments. Due to space constraints, we only report the correlation matrix and portfolio return statistics with a one-month holding period. All other results are available on request. Table 9 shows a low correlation among our variables, except for a high negative correlation between implied skewness and skewness risk premium.

INSERT TABLE 9 HERE

Table 10 highlights the strong cross-sectional findings for the implied skewness, and variance risk premium together with those for the double sorting strategies. These results confirm the previous ones from Section IV and Table 7.

INSERT TABLE 10 HERE

We next investigate whether the profitability of realized and implied moments and their risk premia is solely a compensation due to exposure to benchmark factors such as equally weighted portfolio, momentum, and roll yield factors. Results of these benchmark strategies in Table 6 show that their returns are not statistically significant. Thus, using them in a factor regression may not make sense. Instead, when using their innovations, the performance of roll yield is the only one not statistically significant.

Nevertheless, in Panels A and B of Table 11, we present the alpha of these strategies using both original variables and their ARMA (1, 1) innovations. In Panel A, we observe that coefficients of the roll yield are negatively significant, and that alpha of implied skewness and SRP is no longer significant. Therefore, we have a surprising situation where, although the roll yield strategy itself is not statistically significant, it helps to explain the predictability of our skewness strategies.

INSERT TABLE 11 HERE

A possible explanation for these findings might be related to the backwardation characteristics of its portfolio. Indeed, examining the strategies on the second and third quartiles in the Appendix, note from Tables A.1 and A.2 that although the roll yield strategy is significant, it cannot explain our trading strategies. Moreover, Figure A.1 from the Appendix points out that for implied skewness, skewness risk premium, and variance risk premium, the mean of the first and fourth quartile returns is positive and negative, respectively. As such, although the mean of all four quartile returns is insignificant, our trading strategies going long and short on the first and fourth quartile are significant and thus, provide reliable findings. The characteristics of the four quartile portfolios have not been reported due to their not statistically significant performance. When using innovations, Panel B also shows the negatively significant roll yield strategy. However, the alphas of implied skewness and double sorting portfolios are significant and, thus, the roll yield does not explain our strategies.

Given that few commodities could consistently enter the long and short portfolios and thus could drive their returns, Figure 2 displays their frequency. Specifically, it shows the frequency of commodities in the long and short portfolios for each of our realized, implied, and risk premium moments, as well as that of the momentum and roll yield. Note that the roll yield portfolio favors commodities with the highest positive and lowest negative roll yields without giving much attention to other commodities. In particular, most times, we would enter a long position on corn and soybeans commodities and a short position on wheat and natural gas commodities. As such, we once again confirm that after 2012, the backwardation of commodities affects our results, and thus we obtain a negative return for the roll yield portfolio in Table 6. In general, commodity frequencies are way below 50%, and therefore, over time, various commodities enter the long and short portfolios of our variables.

INSERT FIGURE 2 HERE

We further consider additional multivariate regressions as in Section III that include several other control variables. Taking, for instance, the realized variance's predictability in Equation (1), we also add as a control variable the commodity's individual realized skewness and kurtosis. For the realized skewness's predictability, we control for the commodity's individual realized variance and kurtosis. We estimate similar regressions for the implied moments and risk premia shown in Tables 4 and 5. In addition to the one-month return, we forecast the second-month and third-month returns. Moreover, during our analyses, we also consider the one-month momentum and roll yield. Furthermore, we control for the Fama and French (1993) factors and the momentum factor. In all cases, the results are very similar. Due to space constraints, we do not report these results but are available on request.

We also analyze the usefulness of our portfolios from the viewpoint of a diversified U.S. investor by building the Markowitz efficient frontier. Figure A.2 from the Appendix informs that the best risk-return tradeoff belongs to the SRP and implied skewness portfolios and bond market. In addition, Table A.6 from Appendix shows that two of our portfolios – SRP and implied skewness – prove beneficial from an optimization's viewpoint, with relevant risk-return tradeoff and weights in both low and high volatility portfolios.

Finally, we examine the role of realized and implied kurtosis and kurtosis risk premium in predicting commodity returns. Generally, these findings are not statistically significant, and thus, we do not present them, but they are available on request.[9]

VI. Conclusion

This paper provides new empirical evidence on the relationship between commodity futures returns and realized, implied, and risk premium moments. To estimate the realized and implied variance and skewness, we use high frequency and options on futures data, respectively. We define the risk premia as the difference between the realized and implied moments.

We shed light on the predictive ability of implied skewness and skewness risk premium from both a time-series and cross-sectional perspective. Specifically, we highlight the superior predictive power of implied skewness over that of realized variance and skewness, as well as the implied variance and variance risk premium from both perspectives. These findings suggest information differences between commodity derivative markets. Moreover, we show that a long and short

[9] The study of Nguyen and Prokopczuk (2019) documents the existence of high jump co-movements in commodity markets. However, the authors emphasize that these occur much less frequently than in stock markets which may to some extent explain the insignificant results.

position on the portfolio with the highest and respectively the lowest implied skewness and skewness risk premium yields an average return of around 17% and 18%, with a volatility of around 28% and 26%. Furthermore, from a U.S. investor's viewpoint, these portfolios seem worthwhile, exhibiting good risk-return tradeoff and relevant weights in the efficient frontier portfolios with risk similar to bond, equity, and commodity markets.

Our findings are economically significant, and factors such as the momentum and roll yield partially explain them. All taken together, the time-series and cross-sectional results are robust to other various controls and portfolio selection measures.

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TABLES

Table 1 - Summary Statistics

This table presents summary statistics for each of the commodities covering the period from January 2008 to December 2016. It provides the mean, standard deviation (Std. Dev.), 25th quantile, and 75th quantile. Panels A and B show these statistics for the realized and implied variance and skewness, and Panel C presents them for the variance and skewness risk premia. We follow Amaya et al. (2015) and Bakshi et al. (2003) to compute the daily realized variance and skewness from the intraday 5-minute returns and their respective implied moments from daily options data. In particular, we compute the realized moments on a daily overlapping basis of monthly magnitude. Moment risk premia are defined as the difference between the implied and realized moments (Bollerslev et al., 2009).

Panel A. Realized Moments

	Variance				Skewness			
	Mean	Std. Dev.	25%	75%	Mean	Std. Dev.	25%	75%
Corn	0.007	0.004	0.004	0.010	0.13	1.86	-0.73	1.17
Soybeans	0.006	0.004	0.003	0.009	-0.18	2.16	-1.34	1.16
Wheat	0.013	0.008	0.007	0.018	0.24	1.09	-0.43	0.87
Copper	0.008	0.008	0.004	0.009	-0.46	3.46	-2.57	1.47
Silver	0.008	0.005	0.005	0.010	-0.19	3.42	-2.32	1.72
Gold	0.003	0.003	0.002	0.004	-0.41	3.82	-2.17	1.86
Oil	0.012	0.013	0.004	0.013	-0.42	2.32	-1.79	1.01
Natural Gas	0.014	0.007	0.009	0.020	-0.10	2.70	-1.93	1.75

Panel B. Implied Moments

	Variance				Skewness			
	Mean	Std. Dev.	25%	75%	Mean	Std. Dev.	25%	75%
Corn	0.010	0.008	0.005	0.014	0.32	0.77	-0.16	0.73
Soybeans	0.006	0.005	0.003	0.008	-0.21	0.82	-0.57	0.29
Wheat	0.009	0.006	0.005	0.013	0.49	0.69	0.07	0.96
Copper	0.018	0.019	0.005	0.022	-0.34	1.25	-1.15	0.35
Silver	0.009	0.006	0.005	0.011	-0.11	0.35	-0.33	0.07
Gold	0.004	0.003	0.002	0.004	-0.20	0.43	-0.43	0.07
Oil	0.013	0.011	0.007	0.016	0.01	0.50	-0.29	0.21
Natural Gas	0.014	0.007	0.009	0.018	0.40	0.22	0.27	0.51

Panel C. Moment Risk Premia

	Variance				Skewness			
	Mean	Std. Dev.	25%	75%	Mean	Std. Dev.	25%	75%
Corn	0.003	0.006	0.000	0.004	0.18	2.09	-0.98	1.30
Soybeans	0.000	0.004	-0.002	0.002	-0.03	2.30	-1.42	1.39
Wheat	-0.003	0.008	-0.006	0.001	0.25	1.30	-0.57	1.07
Copper	0.011	0.018	0.001	0.014	0.13	3.82	-2.25	2.54
Silver	0.001	0.004	-0.001	0.002	0.07	3.39	-1.85	2.09
Gold	0.000	0.002	0.000	0.001	0.21	3.79	-1.94	1.87
Oil	0.002	0.006	0.000	0.004	0.43	2.30	-1.03	1.81
Natural Gas	0.000	0.005	-0.003	0.002	0.50	2.74	-1.40	2.39

Table 2 – Realized and Implied Moments and Risk Premia's Predictability

This table shows the results of 48 regressions, $\mathrm{Ret}_{i,t,h} = \alpha + \beta_1 * \mathrm{M}_{i,t-1} + \beta_2 * \mathrm{Ret}_{i,t-h,h} + \varepsilon_{i,t}$, where $\mathrm{M}_{i,t-1}$ is the realized, implied, and risk premium moment of commodity *i* at time *t-1* and $\mathrm{Ret}_{i,t-h,h}$ is commodity *i* return for the window *t* - *h* to *t*. The dependent variables are the returns of the commodity futures for h being one-month. Each regression has as independent variables each of the realized and implied moment and its risk premium of commodity returns using five-minute returns, the lagged dependent variable, and a constant. All the independent variables are lagged. In Panel A, the independent variables are each of the realized variance and skewness, in Panel B, are the implied variance and skewness, and Panel C shows their risk premia. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The *t*-statistics are Hansen-Hodrick HAC with h+1 lags, where *h* is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Individual Realized Moments

Dependent Variable	Coefficients		Adjusted R^2	
	Realized Variance	Realized Skewness		
Corn	-1.13	0.005	0.4%	0.9%
	(-0.46)	(1.04)		
Soybeans	-1.95	-0.003	1.0%	0.4%
	(-1.02)	(-0.92)		
Wheat	-0.92	-0.004	1.1%	0.8%
	(-0.78)	(-0.64)		
Copper	0.70	-0.0003	3.8%	3.4%
	(0.49)	(-0.22)		
Silver	0.58	-0.001	0.1%	0.1%
	(0.22)	(-0.53)		
Gold	0.12	0.001*	2.4%	3.3%
	(0.04)	(1.68)		
Oil	1.04	-0.001	4.9%	3.5%
	(0.94)	(-0.20)		
Natural gas	-0.93	-0.003	0.6%	0.6%
	(-0.45)	(-0.80)		

Panel B. Individual Implied Moments

Dependent Variable	Coefficients		Adjusted R^2	
	Implied Variance	Implied Skewness		
Corn	-1.75	0.02***	2.1%	3.7%
	(-1.56)	(2.44)		
Soybeans	-1.12	0.01*	0.5%	2.0%
	(-0.60)	(1.93)		
Wheat	-0.55	0.03***	0.7%	4.3%
	(-0.37)	(2.78)		
Copper	1.09***	-0.01	9.0%	4.0%
	(2.56)	(-1.07)		
Silver	1.59	0.01	0.9%	0.1%
	(0.89)	(0.34)		
Gold	0.65	0.02*	2.5%	4.3%
	(0.25)	(1.93)		
Oil	1.20	0.04*	4.9%	6.1%
	(1.11)	(1.81)		
Natural gas	1.52	0.14***	0.9%	5.1%
	(0.47)	(2.77)		

Table 2 (continued) – Realized, Implied, and Risk Premium Moment's Predictability

Panel C. Individual Risk Premium Moments

Dependent Variable	Coefficients		Adjusted R^2	
	Variance Risk Premium	Skewness Risk Premium		
Corn	-2.05*	-0.0002	2.0%	0.2%
	(-1.71)	(-0.04)		
Soybeans	0.46	0.004*	0.1%	1.3%
	(0.31)	(1.68)		
Wheat	0.64	0.01*	0.8%	2.6%
	(0.66)	(1.80)		
Copper	1.20***	-0.0004	8.9%	3.4%
	(2.87)	(-0.28)		
Silver	3.17	-0.0002	1.3%	0.1%
	(1.64)	(0.61)		
Gold	2.23	-0.001	2.7%	3.0%
	(0.78)	(-1.40)		
Oil	-0.86	0.003	3.7%	3.8%
	(-0.36)	(0.90)		
Natural gas	5.31	0.004	3.5%	0.8%
	(1.39)	(1.07)		

Table 3 - Realized Moment's Predictability with Control Variables

This table shows the results of 16 regressions, $Ret_{i,t,h} = \alpha + \beta_1 * RM_{i,t-1} + \beta_2 * RY_{i,t-1} + \beta_3 * DXY_{i,t-1} + \beta_4 * Ret_{i,t-h,h} + \varepsilon_{i,t}$, where $RM_{i,t-1}$ is the realized moment of commodity *i* at time *t-1* and $Ret_{i,t-h,h}$ is commodity *i* return for the window *t* - *h* to *t*, $RY_{i,t-1}$ is the roll yield of commodity *i* at time *t-1*, $DXY_{i,t-1}$ is the dollar index return at time *t-1*. The dependent variables are the returns of the commodity futures for h = one month. Each regression has as independent variables the realized moment of our eight commodities, the roll yield of the commodity, the dollar index previous month returns, and the one-month lagged dependent variable and a constant. All the independent variables are lagged. Realized moments are calculated using five-minute returns on the previous one month. In Panel A, the main independent variable is the realized variance, and in Panel B is the realized skewness. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The *t*-statistics are Hansen-Hodrick HAC with h+1 lags, where h is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Realized Variance

Dependent Variable	Coefficients				Adjusted R^2
	Realized Variance	Roll Yield	Dollar Index	Lagged Returns	
Corn	-1.50	-0.12	-0.52*	-0.06	2.9%
	(-0.64)	(-1.49)	(-1.90)	(-0.69)	
Soybeans	-2.24	-0.25***	-0.57***	-0.03	6.5%
	(-1.27)	(-2.37)	(-2.47)	(-0.37)	
Wheat	-1.14	-0.38**	-0.21	-0.01	5.2%
	(-1.04)	(-1.99)	(-0.56)	(-0.16)	
Copper	0.06	-1.26**	-0.32	0.20*	8.5%
	(0.05)	(-2.29)	(-1.39)	(1.84)	
Silver	0.57	3.38**	-0.27	-0.06	5.6%
	(0.24)	(2.14)	(-1.08)	(-0.80)	
Gold	1.32	1.23	-0.06	-0.16*	4.1%
	(0.45)	(1.38)	(-0.40)	(-1.66)	
Oil	-0.82	-0.37*	-0.56*	0.20**	10.4%
	(-0.51)	(-1.88)	(-1.96)	(2.07)	
Natural gas	-4.82***	-0.34***	-0.11	0.03	10.9%
	(-2.95)	(-2.71)	(-0.30)	(0.49)	

Panel B. Realized Skewness

Dependent Variable	Coefficients				Adjusted R^2
	Realized Skewness	Roll Yield	Dollar Index	Lagged Returns	
Corn	0.01	-0.12	-0.58*	-0.08	3.6%
	(1.30)	(-1.52)	(-1.95)	(-1.04)	
Soybeans	0.00	-0.24**	-0.58***	0.03	5.8%
	(-1.06)	(-2.21)	(-2.45)	(0.29)	
Wheat	0.00	-0.36*	-0.24	-0.02	4.4%
	(-0.15)	(-1.92)	(-0.64)	(-0.27)	
Copper	0.00	-1.30***	-0.32	0.22*	8.6%
	(-0.75)	(-2.40)	(-1.40)	(1.75)	
Silver	0.00	3.39**	-0.25	-0.05	5.5%
	(-0.22)	(2.18)	(-0.96)	(-0.71)	
Gold	0.00*	1.01	-0.02	-0.20*	4.7%
	(1.69)	(1.22)	(-0.16)	(-1.90)	
Oil	0.00	-0.30***	-0.64**	0.23**	10.4%
	(-1.04)	(-2.77)	(-2.07)	(2.07)	
Natural gas	0.001	-0.23*	-0.23	-0.03	6.8%
	(0.39)	(-1.89)	(-0.62)	(-0.34)	

Table 4 - Implied Moment's Predictability with Control Variables

This table shows the results of 16 regressions, $Ret_{i,t,h} = \alpha + \beta_1 * IM_{i,t-1} + \beta_2 * RY_{i,t-1} + \beta_3 * DXY_{i,t-1} + \beta_4 * Ret_{i,t-h,h} + \varepsilon_{i,t}$, where $IM_{i,t-1}$ is the implied moment of commodity *i* at time *t-1* and $Ret_{i,t-h,h}$ is commodity *i* return for the window *t - h* to *t*, $RY_{i,t-1}$ is the roll yield of commodity *i* at time *t-1*, $DXY_{i,t-1}$ is the dollar index return at time *t-1*. The dependent variables are the returns of the commodity futures for one month. Each regression has as independent variables the implied moment of our eight commodities, the roll yield of the commodity, the dollar index previous month returns, and the one-month lagged dependent variable and a constant. All the independent variables are lagged. Implied moments are calculated using one-month options on the previous one month. In Panel A, the main independent variable is the implied variance and in Panel B, is the implied skewness. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The *t*-statistics are Hansen-Hodrick HAC with h+1 lags, where h is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Implied Variance

Dependent Variable	Coefficients				Adjusted R^2
	Implied Variance	Roll Yield	Dollar Index	Lagged Returns	
Corn	-1.94*	-0.13*	-0.50*	-0.06	4.79%
	(-1.83)	(-1.67)	(-1.78)	(-0.72)	
Soybeans	-1.04	-0.23**	-0.58***	-0.01	5.54%
	(-0.59)	(-2.16)	(-2.54)	(-0.08)	
Wheat	-1.08	-0.38*	-0.27	-0.02	4.80%
	(-0.76)	(-1.92)	(-0.73)	(-0.17)	
Copper	0.94***	-1.06**	-0.28	0.14	12.48%
	(2.43)	(-2.23)	(-1.31)	(1.07)	
Silver	1.17	3.23**	-0.30	-0.06	5.97%
	(0.73)	(1.97)	(-1.14)	(-0.83)	
Gold	1.94	1.32	-0.08	-0.17*	4.54%
	(0.75)	(1.56)	(-0.59)	(-1.93)	
Oil	-1.26	-0.40**	-0.54*	0.19**	10.68%
	(-0.76)	(-2.05)	(-1.96)	(2.14)	
Natural gas	-3.57	-0.33***	-0.18	0.01	8.35%
	(-1.15)	(-2.80)	(-0.48)	(0.10)	

Panel B. Implied Skewness

Dependent Variable	Coefficients				Adjusted R^2
	Implied Skewness	Roll Yield	Dollar Index	Lagged Returns	
Corn	0.02**	-0.06	-0.53**	-0.03	5.21%
	(2.30)	(-0.81)	(-1.97)	(-0.41)	
Soybeans	0.01	-0.20*	-0.57**	-0.01	5.61%
	(0.92)	(-1.68)	(-2.26)	(-0.11)	
Wheat	0.03***	-0.31*	-0.27	-0.02	7.23%
	(3.01)	(-1.82)	(-0.72)	(-0.18)	
Copper	-0.01*	-1.45***	-0.31	0.17	10.15%
	(-1.82)	(-2.53)	(-1.37)	(1.48)	
Silver	0.02	3.52**	-0.26	-0.07	5.89%
	(0.72)	(2.19)	(-1.01)	(-0.95)	
Gold	0.02**	1.08	-0.06	-0.22**	5.93%
	(2.18)	(1.24)	(-0.46)	(-2.29)	
Oil	0.00	-0.27**	-0.58*	0.20**	10.04%
	(0.17)	(-2.02)	(-1.95)	(2.01)	
Natural gas	0.10***	-0.18	-0.22	-0.01	8.81%
	(2.45)	(-1.60)	(-0.59)	(-0.20)	

Table 5 - Moment Risk Premium's Predictability with Control Variables

This table shows the results of 16 regressions, $\mathrm{Ret}_{i,t,h} = \alpha + \beta_1 * \mathrm{MRP}_{i,t-1} + \beta_2 * \mathrm{RY}_{i,t-1} + \beta_3 * \mathrm{DXY}_{i,t-1} + \beta_4 * \mathrm{Ret}_{i,t-h,h} + \varepsilon_{i,t}$, where $\mathrm{MRP}_{i,t-1}$ is the moment risk premium of commodity *i* at time *t-1* and $\mathrm{Ret}_{i,t-h,h}$ is commodity *i* return for the window *t* - *h* to *t*, $\mathrm{RY}_{i,t-1}$ is the roll yield of commodity *i* at time *t-1*, $\mathrm{DXY}_{i,t-1}$ is the dollar index return at time *t-1*. The dependent variables are the returns of the commodity futures for one month. Each regression has as independent variables the moment risk premium of our eight commodities, the roll yield of the commodity, the dollar index previous month returns, and the one-month lagged dependent variable and a constant. All the independent variables are lagged. Moment risk premia are calculated using one-month options and five-minute returns on the previous one month. In Panel A, the independent variable is the variance risk premium and in Panel B, is the skewness risk premium. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The *t*-statistics are Hansen-Hodrick HAC with h+1 lags, where h is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Variance Risk Premium

Dependent Variable	Coefficients				Adjusted R^2
	Variance Risk Premium	Roll Yield	Dollar Index	Lagged Returns	
Corn	-2.13*	-0.11	-0.54*	-0.06	4.47%
	(-1.87)	(-1.51)	(-1.88)	(-0.74)	
Soybeans	0.86	-0.24**	-0.60***	-0.02	5.33%
	(0.55)	(-2.14)	(-2.43)	(-0.18)	
Wheat	0.55	-0.36*	-0.21	-0.02	4.56%
	(0.57)	(-1.91)	(-0.55)	(-0.28)	
Copper	1.12***	-1.17**	-0.28	0.12	13.20%
	(3.00)	(-2.32)	(-1.30)	(1.03)	
Silver	2.07	3.16*	-0.28	-0.06	6.05%
	(1.01)	(1.93)	(-1.03)	(-0.90)	
Gold	2.22	0.99	-0.05	-0.18*	4.04%
	(0.77)	(1.20)	(-0.33)	(-1.75)	
Oil	-0.02	-0.29***	-0.58**	0.20*	10.01%
	(-0.01)	(-2.59)	(-1.96)	(1.74)	
Natural gas	3.97	-0.20*	-0.21	0.00	8.43%
	(1.19)	(-1.90)	(-0.55)	(-0.02)	

Panel B. Skewness Risk Premium

Dependent Variable	Coefficients				Adjusted R^2
	Skewness Premium	Roll Yield	Dollar Index	Lagged Returns	
Corn	0.00	-0.11	-0.56*	-0.07	2.67%
	(-0.40)	(-1.40)	(-1.91)	(-0.81)	
Soybeans	0.00	-0.22**	-0.57***	0.03	6.08%
	(1.43)	(-2.09)	(-2.37)	(0.33)	
Wheat	0.01	-0.31*	-0.22	-0.01	5.40%
	(1.60)	(-1.82)	(-0.58)	(-0.09)	
Copper	0.00	-1.26***	-0.32	0.20	8.46%
	(-0.12)	(-2.37)	(-1.42)	(1.62)	
Silver	0.00	3.38**	-0.25	-0.05	5.53%
	(0.37)	(2.19)	(-0.95)	(-0.67)	
Gold	0.00	1.01	-0.02	-0.19*	4.36%
	(-1.39)	(1.22)	(-0.16)	(-1.81)	
Oil	0.00	-0.29***	-0.65**	0.23**	10.44%
	(1.13)	(-2.64)	(-2.10)	(2.14)	
Natural gas	0.00	-0.23*	-0.24	-0.02	6.76%
	(-0.16)	(-1.85)	(-0.64)	(-0.25)	

Table 6 - Portfolio Return Statistics

This table presents the portfolio return statistics for the one-month (21 days) holding period. The mean and standard deviation are annualized, and ***, **, * indicate the significance at 1%, 5%, and 10% level, respectively. The sample period is from 2008 to 2016, on a daily overlapping basis.

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
Realized Variance	-1.77%	32.7%	0.72	8.6	-0.05	-0.17
Realized Skewness	-9.40%	25.2%	-0.43	6.9	-0.37	-1.13
Implied Variance	4.59%	31.5%	0.54	7.2	0.15	0.46
Implied Skewness	17.21%	28.0%	0.84	7.3	0.62	1.66*
Variance Risk Premium	12.30%	25.4%	0.08	5.2	0.48	1.38
Skewness Risk Premium	18.37%	25.9%	0.62	7.7	0.71	1.88*
Equally-Weighted Returns	2.31%	21.7%	-0.03	7.5	0.11	0.31
1 Year Momentum	-15.66%	36.5%	-0.65	6.6	-0.43	-1.58
1 Year Roll Yield	-11.21%	34.9%	-0.80	7.1	-0.32	-1.03

Table 7 - Portfolio Return Statistics using Double Sorting

This table presents the portfolio return statistics using double sorting and a one-month holding period (21 days). The mean and standard deviation are annualized, and ***, **, * indicate the significance at 1%, 5%, and 10% level, respectively. The sample period is from 2008 to 2016, on a daily overlapping basis.

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
VRP and Implied Skewness	13.10%	22.1%	0.863	9.1	0.59	1.66*
VRP and SRP	16.44%	22.1%	0.883	10.4	0.75	1.98**
SRP and Implied Skewness	17.94%	27.6%	1.038	9.4	0.65	1.80*

Table 8 - Portfolio Return Statistics using Different Holding Periods

This table presents the portfolio return statistics based on single and double sorting using various holding periods. Panel A shows the portfolios statistics for the two-week holding period (10 days), and Panel B shows them for the two-month holding period (42 days). The mean and standard deviation are annualized, and ***, **, * indicate the significance at 1%, 5%, and 10% level, respectively. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Two-Week Holding Period

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
Realized Variance	-1.97%	34.3%	0.646	7.7	-0.06	-0.18
Realized Skewness	-7.49%	28.5%	-0.325	6.1	-0.26	-0.79
Implied Variance	7.81%	32.7%	0.539	7.2	0.24	0.73
Implied Skewness	18.87%	29.6%	0.907	7.0	0.64	1.74*
Variance Risk Premium	20.80%	28.3%	0.076	5.6	0.73	2.12**
Skewness Risk Premium	18.98%	29.1%	0.381	5.8	0.65	1.75*
Equally-Weighted Returns	2.70%	21.6%	-0.030	7.5	0.12	0.36
One-Year Momentum	-15.49%	37.5%	-0.587	6.4	-0.41	-1.46
One-Year Roll Yield	-9.14%	35.5%	-0.757	7.0	-0.26	-0.83
VRP and Implied Skewness	19.88%	24.4%	0.635	6.4	0.81	2.25**
VRP and SRP	23.29%	25.3%	0.955	10.2	0.92	2.50***
SRP and Implied Skewness	15.99%	29.9%	1.036	8.4	0.53	1.52

Panel B. Two-Month Holding Period

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
Realized Variance	3.92%	31.3%	0.788	9.0	0.13	0.41
Realized Skewness	-2.34%	21.1%	-0.623	9.6	-0.11	-0.29
Implied Variance	6.03%	30.1%	0.684	8.4	0.20	0.71
Implied Skewness	12.27%	26.2%	0.767	7.3	0.47	1.27
Variance Risk Premium	7.34%	22.4%	0.298	5.9	0.33	0.98
Skewness Risk Premium	9.49%	22.4%	0.890	11.4	0.42	1.11
Equally-Weighted Returns	2.12%	21.7%	-0.026	7.5	0.10	0.27
1 Year Momentum	-17.45%	34.9%	-0.606	6.5	-0.50	-1.92*
1 Year Roll Yield	-15.35%	33.8%	-0.848	7.4	-0.45	-1.57
VRP and Implied Skewness	8.22%	19.9%	0.814	10.6	0.41	1.26
VRP and SRP	8.86%	19.1%	0.880	13.5	0.46	1.24
SRP and Implied Skewness	12.02%	24.9%	1.057	10.5	0.48	1.34

Table 9 - Portfolio Correlation Matrix using Innovations

This table presents the portfolio return correlations among realized, implied, and risk premium moments, as well as the one-year momentum and roll yield. These portfolios are built using single sorting ARMA (1, 1) innovations of previous variables for a one-month holding period. The sample period is from 2008 to 2016, on a daily overlapping basis.

	Realized Variance	Realized Skewness	Implied Variance	Implied Skewness	VRP	SRP	Equally-Weighted	Momentum	Roll Yield
Realized Variance									
Realized Skewness	-4%								
Implied Variance	27%	-2%							
Implied Skewness	15%	-10%	5%						
VRP	-34%	8%	51%	-8%					
SRP	8%	-84%	7%	37%	-10%				
Equally-Weighted	3%	-2%	1%	-1%	-1%	-1%			
Momentum	-4%	30%	-16%	-21%	-2%	-36%	0%		
Roll Yield	-9%	27%	-16%	-33%	-1%	-35%	-2%	46%	

Table 10 - Portfolio Return Statistics using Innovations

This table presents the portfolio return statistics using single and double sorting approaches. These portfolios are built using the ARMA (1, 1) commodity innovations with a one-month holding period. The mean and standard deviation are annualized, and ***, **, * indicate the significance at 1%, 5%, and 10% level, respectively. The sample period is from 2008 to 2016, on a daily overlapping basis.

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
Realized Variance	-5.21%	11.5%	0.117	6.5	-0.45	-1.44
Realized Skewness	-0.21%	11.6%	-0.323	6.9	-0.02	-0.06
Implied Variance	2.59%	9.1%	-0.206	10.2	0.29	0.87
Implied Skewness	6.15%	9.7%	1.226	11.4	0.64	1.89*
Variance Risk Premium	5.96%	9.7%	-0.279	10.9	0.61	1.64*
Skewness Risk Premium	1.66%	9.7%	0.362	6.9	0.17	0.55
Equally-Weighted Returns	2.31%	21.7%	-0.028	7.5	0.11	0.31
1 Year Momentum	-1.61%	8.1%	-0.141	9.0	-0.20	-0.59
1 Year Roll Yield	12.00%	9.6%	-1.097	14.0	1.25	3.69***
VRP and Implied Skewness	8.77%	8.9%	0.754	7.1	0.98	2.78***
VRP and SRP	7.40%	9.0%	0.457	6.8	0.82	2.28**
SRP and Implied Skewness	5.43%	9.0%	1.055	10.6	0.60	1.85*

Table 11 - Factor Regressions

This table presents the performance of commodity portfolios. Panel A shows the performance for the portfolios built using the realized and implied moments and their risk premia, and Panel B for the portfolios built using their ARMA (1, 1) innovations. We use a one-month holding period. The momentum and roll yield variables are the one-year momentum and one-year roll yield. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Original Variables

Dependent Variable	Coefficients				Adjusted R^2
	Alfa	Equal-Weighted	Moment	Roll Yield	
Realized Variance	-0.0004	-0.03*	-0.06	-0.54***	38.4%
	(-1.05)	(-1.65)	(-0.54)	(-8.06)	
Realized Skewness	-0.0002	-0.03	0.21***	0.06	12.8%
	(-0.69)	(-1.41)	(3.93)	(1.15)	
Implied Variance	-0.0001	-0.05***	-0.09	-0.43***	30.0%
	(-0.25)	(-2.75)	(-0.95)	(-7.91)	
Implied Skewness	0.0004	0.05	-0.02	-0.45***	33.0%
	(1.23)	(1.62)	(-0.27)	(-6.56)	
Variance Risk Premium	0.0005	-0.03	-0.06	0.13**	2.0%
	(1.50)	(-1.30)	(-0.93)	(2.07)	
Skewness Risk Premium	0.0004	0.05*	-0.21***	-0.16***	21.8%
	(1.49)	(1.78)	(-4.03)	(-2.93)	
VRP and Implied Skewness	0.0004	0.016	-0.033	-0.220***	14.6%
	(1.37)	(0.84)	(-0.57)	(-4.88)	
VRP and SRP	0.0005*	0.016	-0.156***	-0.043	9.3%
	(1.66)	(0.87)	(-3.35)	(-0.89)	
SRP and Implied Skewness	0.0004	0.051*	-0.132**	-0.378***	36.3%
	(1.31)	(1.66)	(-1.98)	(-5.99)	

Panel B. Innovations

Dependent Variable	Coefficients				Adjusted R^2
	Alfa	Equal-Weighted	Moment	Roll Yield	
Realized Variance	-0.0002	0.017	-0.005	-0.104	0.9%
	(-1.14)	(1.16)	(-0.05)	(-1.39)	
Realized Skewness	-0.0001	-0.009	0.330***	0.194**	11.4%
	(-0.60)	(-0.80)	(3.03)	(2.14)	
Implied Variance	0.0001	0.003	-0.125	-0.107	3.7%
	(1.13)	(0.34)	(-1.34)	(-1.30)	
Implied Skewness	0.0004***	-0.006	-0.094	-0.297***	11.4%
	(2.52)	(-0.72)	(-0.87)	(-2.97)	
Variance Risk Premium	0.0002	-0.004	-0.018	-0.001	0.0%
	(1.47)	(-0.49)	(-0.18)	(-0.01)	
Skewness Risk Premium	0.0002	-0.005	-0.303***	-0.241***	17.4%
	(1.52)	(-0.49)	(-3.27)	(-3.72)	
VRP and Implied Skewness	0.0004***	0.006	-0.079	-0.184***	5.7%
	(3.62)	(0.73)	(-1.05)	(-3.29)	
VRP and SRP	0.0003***	-0.001	-0.161**	-0.143***	6.4%
	(2.87)	(-0.08)	(-1.98)	(-2.65)	
SRP and Implied Skewness	0.0003***	-0.009	-0.179*	-0.294***	17.1%
	(2.80)	(-0.94)	(-1.94)	(-4.00)	

FIGURES

Figure 1 - Portfolio Path

This figure presents the portfolio path of single-sort realized, implied, and risk premium moment portfolios (variance and skewness) considering a one-month (21 days) holding period. The sample period is from 2008 to 2016, on a daily overlapping basis.

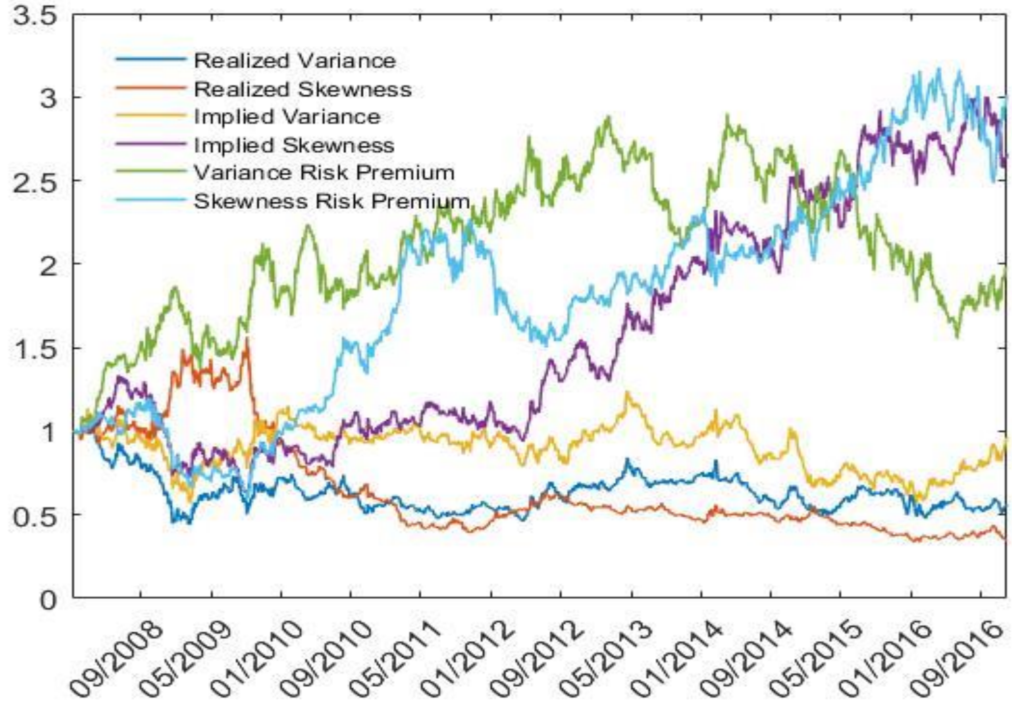
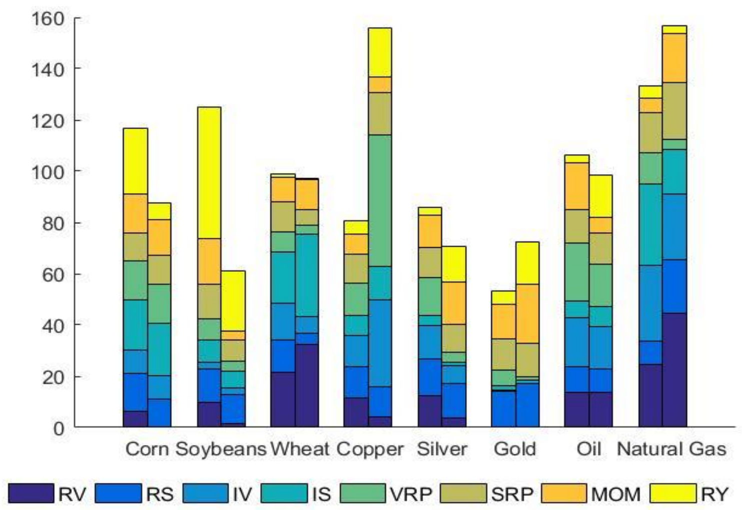


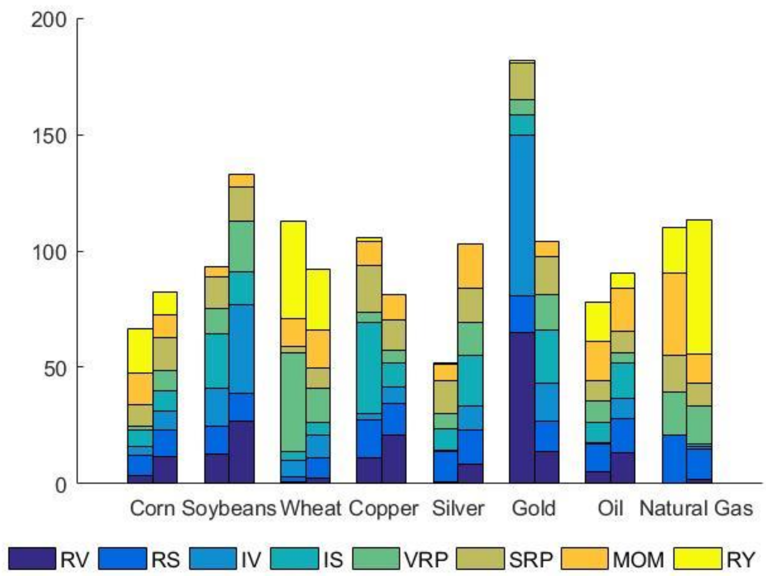
Figure 2 - Frequency of Commodities in Portfolio Analysis

This figure shows the percentage of days over the entire sample period from January 2008 to December 2016. In particular, when each commodity enters the long portfolio (Panel A) and short portfolio (Panel B) for each of the realized and implied moments and risk premium moments, as well as for the one-year momentum and roll yield portfolios. The RV and RS are the realized variance and skewness, and the IV and IS are the implied variance and skewness. VRP and SRP are the variance and skewness risk premia. MOM and RY are the momentum and roll yield, respectively.

Panel A. Long Portfolios



Panel B. Short Portfolios



Appendix

Table A.1 - Portfolio Return Statistics – Second and Third Quartiles

This table presents the portfolio return statists considering a one-month holding period (21 days) for the long-short portfolio of the second and third quartiles. That is, we present results for portfolios going long in the second quartile and short in the third quartile. The mean and standard deviation are annualized, and ***, **, * indicate the significance at 1%, 5%, and 10% level, respectively. The sample period is from 2008 to 2016, on a daily overlapping basis.

Criteria for Portfolio	Mean	Standard Deviation	Skewness	Kurtosis	Sharpe Ratio	*t*-stat for Returns <> 0
Realized Variance	-0.79%	19.6%	0.359	8.5	-0.04	-0.12
Realized Skewness	-1.25%	15.9%	-0.090	7.6	-0.08	-0.24
Implied Variance	0.63%	16.6%	0.185	6.6	0.04	0.12
Implied Skewness	-4.35%	16.8%	0.646	9.4	-0.26	-0.83
Variance Risk Premium	-0.54%	12.5%	0.272	7.0	-0.04	-0.13
Skewness Risk Premium	0.66%	15.6%	0.234	7.4	0.04	0.15
Equally-Weighted Returns	2.31%	21.7%	-0.028	7.5	0.11	0.31
1 Year Momentum	-4.24%	20.9%	-0.419	7.4	-0.20	-0.69
1 Year Roll Yield	16.64%	23.8%	-0.186	5.4	0.70	1.79*
VRP and Implied Skewness	13.10%	22.1%	0.863	9.1	0.59	1.66*
VRP and SRP	16.44%	22.1%	0.883	10.4	0.75	1.98**
SRP and Implied Skewness	17.94%	27.6%	1.038	9.4	0.65	1.80*

Table A.2 - Factor Regressions for Second and Third Quartiles

This table presents the performance of commodity portfolios built using the realized and implied moments and their risk premia with a one-month holding period. We show the portfolios' performance by going long (short) on the second (third) quartile of realized, implied, and risk premium moments. The momentum and roll yield variables are the one-year momentum and one-year roll yield. The sample period is from 2008 to 2016, on a daily overlapping basis.

Dependent Variable	Coefficients				Adjusted R^2
	Alfa	Equal-Weighted	Momentum	Roll Yield	
Realized Variance	-0.0001	0.021	-0.083	0.033	0.9%
	(-0.27)	(1.11)	(-1.49)	(0.49)	
Realized Skewness	-0.0001	0.017	0.005	0.001	0.1%
	(-0.25)	(0.69)	(0.14)	(0.02)	
Implied Variance	0.0000	-0.008	-0.030	-0.001	0.2%
	(0.09)	(-0.42)	(-0.55)	(-0.01)	
Implied Skewness	-0.0001	0.013	0.039	-0.071	1.1%
	(-0.62)	(0.93)	(0.66)	(-1.61)	
Variance Risk Premium	0.0000	0.006	-0.002	0.005	0.0%
	(-0.18)	(0.60)	(-0.04)	(0.17)	
Skewness Risk Premium	0.0000	-0.001	0.028	-0.027	0.3%
	(0.28)	(-0.03)	(0.61)	(-0.67)	
VRP and Implied Skewness	0.0005*	0.008	-0.036	-0.040	0.3%
	(1.91)	(0.47)	(-0.58)	(-0.72)	
VRP and SRP	0.0006**	0.010	-0.048	-0.038	0.4%
	(2.01)	(0.58)	(-0.74)	(-0.79)	
SRP and Implied Skewness	0.0008**	0.029	-0.055	-0.187***	3.1%
	(2.07)	(0.82)	(-0.68)	(-2.42)	

Table A.3 - Realized Moment's Predictability

This table shows the results of 32 regressions, $\text{Ret}_{i,t,h} = \alpha + \beta_1 * \text{RM}_{i,t-1} + \beta_2 * \text{Ret}_{i,t-h,h} + \varepsilon_{i,t}$, where $\text{RM}_{i,t-1}$ is the realized moment of commodity *i* at time *t-1* and $\text{Ret}_{i,t-h,h}$ is commodity *i* return for the window *t - h* to *t*. The dependent variables are the returns of the commodity futures for h = two and three months. Each regression has the realized moment of commodity returns as an independent variable using five-minute returns, the lagged dependent variable, and a constant. All the independent variables are lagged. In Panel A, the independent variable is the realized variance, and in Panel B, it is the realized skewness. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The *t*-statistics are Hansen-Hodrick HAC with h+1 lags, where *h* is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Individual Realized Variance

Dependent Variable	Coefficients		Adjusted R^2	
	2nd Month	3rd Month	2nd Month	3rd Month
Corn	-1.31	-2.10	0.3%	0.6%
	(-0.31)	(-0.36)		
Soybeans	-2.02	-4.77	0.6%	3.9%
	(-0.56)	(-1.08)		
Wheat	-0.86	-1.20	4.9%	2.1%
	(-0.60)	(-0.73)		
Copper	4.20*	7.81***	10.2%	12.9%
	(1.71)	(2.96)		
Silver	3.42	7.87*	1.4%	4.8%
	(0.84)	(1.94)		
Gold	2.53	5.17	2.1%	2.9%
	(0.67)	(1.24)		
Oil	3.61*	5.65**	9.0%	11.2%
	(1.73)	(1.99)		
Natural gas	0.28	2.96	0.0%	1.2%
	(0.07)	(0.59)		

Panel B. Individual Realized Skewness

Dependent Variable	Coefficients		Adjusted R^2	
	2nd Month	3rd Month	2nd Month	3rd Month
Corn	0.004	0.002	0.4%	0.4%
	(0.53)	(0.32)		
Soybeans	0.003	0.003	0.3%	2.1%
	(0.73)	(0.71)		
Wheat	0.000	0.002	4.6%	1.7%
	(0.00)	(0.15)		
Copper	0.004**	0.005*	5.5%	2.2%
	(2.04)	(1.73)		
Silver	0.000	0.001	0.1%	0.2%
	(0.17)	(0.21)		
Gold	0.000	0.001	1.1%	0.2%
	(0.18)	(0.30)		
Oil	0.001	0.006	1.7%	1.4%
	(0.15)	(0.93)		
Natural gas	-0.004	-0.001	0.3%	0.4%
	(-0.66)	(-0.18)		

Table A.4 - Implied Moment's Predictability

This table shows the results of 32 regressions, $\mathrm{Ret}_{i,t,h} = \alpha + \beta_1 * \mathrm{IM}_{i,t-1} + \beta_2 * \mathrm{Ret}_{i,t-h,h} + \varepsilon_{i,t}$ where $\mathrm{IM}_{i,t-1}$ is the implied moment of commodity *i* at time *t-1* and $\mathrm{Ret}_{i,t-h,h}$ is commodity *i* return for the window *t* - *h* to *t*. The dependent variables are the returns of the commodity futures for h = two and three months. Each regression has as independent variables the implied moments of our eight commodities, the lagged dependent variable, and a constant. All the independent variables are lagged, and the implied moments are calculated using one-month options. In Panel A, the independent variable is implied variance and in Panel B, is the implied skewness. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The *t*-statistics are Hansen-Hodrick HAC with h+1 lags, where h is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Individual Implied Variance

Dependent Variable	Coefficients		Adjusted R^2	
	2nd Month	3rd Month	2nd Month	3rd Month
Corn	-2.30	-2.75	1.8%	1.9%
	(-1.42)	(-1.28)		
Soybeans	-2.10	-4.16	0.9%	3.9%
	(-0.54)	(-0.78)		
Wheat	-1.39	-2.26	5.0%	2.5%
	(-0.80)	(-0.93)		
Copper	2.33***	3.51***	15.8%	16.5%
	(3.12)	(2.90)		
Silver	4.38*	7.24***	3.3%	6.2%
	(1.65)	(2.49)		
Gold	3.56	6.40**	3.0%	4.5%
	(1.17)	(2.02)		
Oil	3.91	5.53	7.9%	8.2%
	(1.62)	(1.41)		
Natural gas	2.43	5.24	0.8%	2.8%
	(0.50)	(0.82)		

Panel B. Individual Implied Skewness

Dependent Variable	Coefficients		Adjusted R^2	
	2nd Month	3rd Month	2nd Month	3rd Month
Corn	0.04***	0.06***	5.7%	8.8%
	(2.75)	(2.96)		
Soybeans	0.02*	0.02	1.9%	3.2%
	(1.66)	(1.43)		
Wheat	0.04***	0.05***	9.3%	6.5%
	(2.97)	(3.10)		
Copper	-0.01	0.00	4.6%	1.2%
	(-0.59)	(-0.05)		
Silver	0.02	0.01	0.2%	0.2%
	(0.32)	(0.18)		
Gold	0.02	0.03	2.8%	1.4%
	(1.23)	(1.03)		
Oil	0.07	0.10	6.1%	6.1%
	(1.52)	(1.36)		
Natural gas	0.17**	0.16*	4.1%	2.9%
	(2.23)	(1.87)		

Table A.5 - Moment Risk Premium's Predictability

This table shows the results of 32 regressions, $\mathrm{Ret}_{i,t,h} = \alpha + \beta_1 * \mathrm{MRP}_{i,t-1} + \beta_2 * \mathrm{Ret}_{i,t-h,h} + \varepsilon_{i,t}$ where $\mathrm{MRP}_{i,t-1}$ is the moment risk premium of commodity *i* at time *t-1* and $\mathrm{Ret}_{i,t-h,h}$ is commodity *i* return for the window *t - h* to *t*. The dependent variables are the returns of the commodity futures for h = two and three months. Each regression has the moment risk premium of our eight commodities, the lagged dependent variable, as independent variables and a constant. All the independent variables are lagged. Moment risk premia are calculated using one-month options and five-minute returns on the previous one month. In Panel A, the independent variable is the variance risk premium and in Panel B, it is the skewness risk premium. The estimates of the constant and lagged dependent variable coefficients are omitted. ***, **, * indicate the significance at 1%, 5% and 10% level, respectively. The *t*-statistics are Hansen-Hodrick HAC with h+1 lags, where h is the size of the return window. The sample period is from 2008 to 2016, on a daily overlapping basis.

Panel A. Individual Variance Risk Premium

Dependent Variable	Coefficients		Adjusted R^2	
	2nd Month	3rd Month	2nd Month	3rd Month
Corn	-2.75*	-3.05*	1.8%	1.7%
	(-1.72)	(-1.94)		
Soybeans	-0.84	-0.60	0.2%	2.0%
	(-0.24)	(-0.16)		
Wheat	0.05	-0.21	4.6%	1.7%
	(0.04)	(-0.17)		
Copper	2.17***	3.06***	12.0%	9.9%
	(3.04)	(2.58)		
Silver	5.45	5.74	2.0%	1.6%
	(1.63)	(1.48)		
Gold	4.36	5.61	1.8%	0.9%
	(0.88)	(0.77)		
Oil	-3.47	-6.05	3.2%	3.9%
	(-0.91)	(-1.28)		
Natural gas	4.39	4.11	1.2%	1.1%
	(0.94)	(0.62)		

Panel B. Individual Skewness Risk Premium

Dependent Variable	Coefficients		Adjusted R^2	
	2nd Month	3rd Month	2nd Month	3rd Month
Corn	0.00	0.01	0.3%	1.0%
	(0.41)	(0.90)		
Soybeans	0.00	0.00	0.1%	1.9%
	(0.09)	(0.00)		
Wheat	0.01	0.01	6.0%	2.7%
	(1.17)	(0.95)		
Copper	0.00**	0.00	5.7%	2.1%
	(-2.02)	(-1.53)		
Silver	0.00	0.00	0.1%	0.2%
	(-0.09)	(-0.19)		
Gold	0.00	0.00	1.1%	0.1%
	(0.00)	(-0.17)		
Oil	0.00	0.00	1.9%	0.9%
	(0.67)	(-0.25)		
Natural gas	0.01	0.00	0.5%	0.5%
	(0.84)	(0.31)		

Table A.6 - Portfolio Weights with the same Risk of Other Assets

This table presents the weights of a Markowitz portfolio optimization estimation. We build an efficient frontier using 11 portfolios: U.S. bond and U.S. equity portfolios, equally-weighted commodity (EW), realized variance (-RV), implied variance (IV), variance risk premium (VRP), realized skewness (-RS), implied skewness (IS), skewness risk premium (SRP), minus one-year momentum (-Momentum) and minus one-year roll yield (-Roll Yield). The estimation period is daily from 2008 to 2016. We do not allow for leverage or short positions. Each column shows the weights of one portfolio on the efficient frontier with specific volatility. For instance, the first column shows the weights of the efficient frontier portfolio with the same volatility as that of the EW commodities' portfolio. The second column shows the weights of the efficient frontier portfolio with the same volatility as that of bonds, and the third column shows the portfolio with the same volatility as that of stocks. We use the S&P 500 and U.S. JP Morgan Global Bond indices as proxies for equity and bond markets and the Barclays three-month T-bill index as a proxy for the risk-free rate.

	Composition of Efficient Frontier Portfolios with		
Weights for	Annual Volatility of 21.55% (**EW Commodity**)	Annual Volatility of 5.05% (**U.S. Bonds**)	Annual Volatility of 22.19% (**U.S. Stocks**)
Realized Variance	0%	0%	0%
- Realized Skewness	0%	0%	0%
Implied Variance	0.0%	0.0%	0.0%
Implied Skewness	28.0%	9%	28%
Variance Risk Premium	3.7%	11.6%	1.5%
Skewness Risk Premium	61.2%	9.3%	63.1%
Equal-Weighted	0%	0%	0%
- Momentum	7.1%	0.6%	7.4%
- Roll Yield	0%	0%	0%
Bonds	0.0%	59%	0%
Stocks	0.0%	11%	0%

Figure A.1 - Portfolio Optimization

This figure shows the Markowitz portfolio optimization. We consider eleven portfolios: U.S. bond and equity portfolios, equally-weighted commodities portfolio (EW), realized variance (-RV), implied variance (IV), variance risk premium (VRP), realized skewness (-RS), implied skewness (IS), skewness risk premium (SRP), minus one-year momentum (-Momentum) and minus one-year roll yield (-Roll Yield). The estimation period is daily from 2008 to 2016. We do not allow for leverage or short positions. The efficient frontier (EF) with a dotted line includes all eleven portfolios, whereas the efficient frontier with a continuous line only includes the U.S. equity and bond markets.

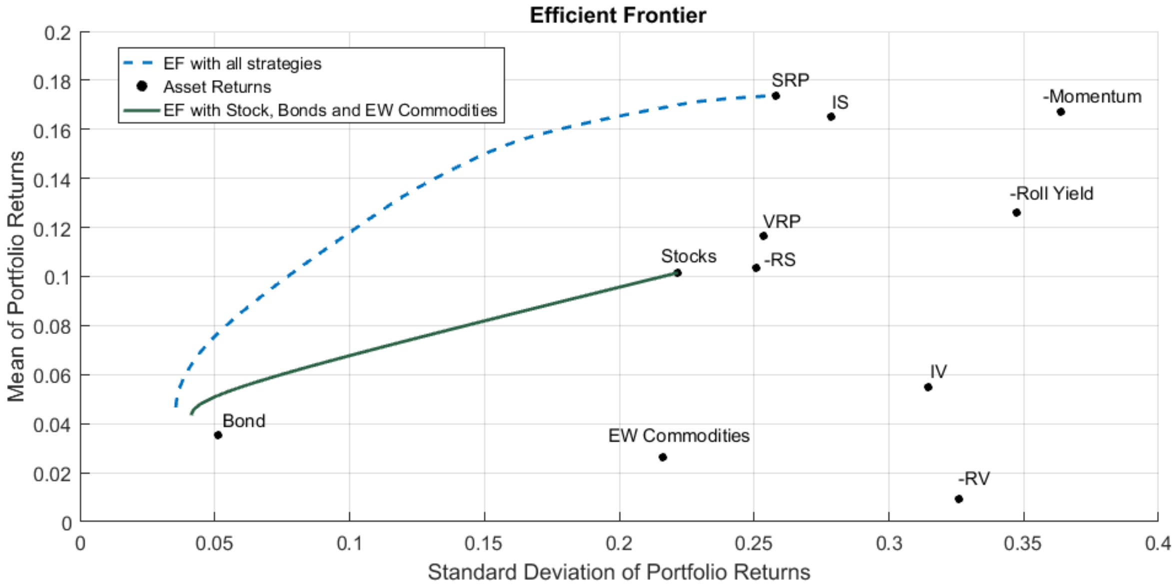


Figure A.2 - Portfolio Mean Return over Quartiles

This figure shows the portfolio mean return for portfolios based on each quartile using a one-month holding period. The means are annualized. RV is the realized variance, and RS is the realized skewness. The IV is the implied variance, and the IS is the implied skewness. VRP is the variance risk premium, and SRP is the skewness risk premium. MOM is the one-year momentum, and RY is the one-year roll yield. The sample period is from 2008 to 2016, on a daily overlapping basis.