The Network Foundations of Credit Counterparty Risk: Theory and Evidence

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Introduction

- **Firm default clustering**
  - Common shocks or correlated risk factors
  - Firm-specific downside shock spillover (i.e. counterparty risk)
- **Counterparty risk**
  - Existing structural frameworks only in highly simplified settings, without explicit economic foundations for the underlying linkages

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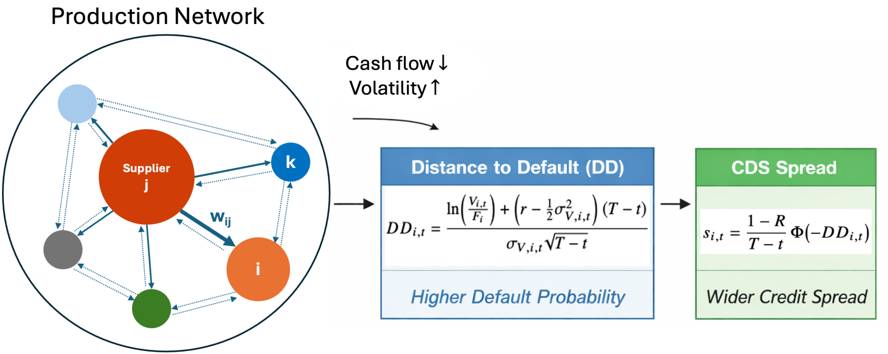
- **This paper**
  - Proposes a structural framework that microfounds counterparty risk within an economy-wide production network
  - Develops a parsimonious empirical approach that captures the role of the full network topology and delivers new predictions for credit spreads

Structural Mapping



CES Production Network

- Firm i produces good i using labor and a CES bundle of intermediate inputs:

$$Y_{i,t} = \xi_t Z_{i,t} L_{i,t}^{1-\alpha_i} \left(\sum_{j=1}^{N} \omega_{ij,t}^{1/\sigma_i} X_{ij,t}^{(\sigma_i-1)/\sigma_i} \right)^{\alpha_i \sigma_i/(\sigma_i-1)} .$$

- $\omega_{ij,t}$ is the primitive input weight of supplier good j in customer firm i's intermediate-input bundle:

$$\sum_{j=1}^{N} \omega_{ij,t} = 1.$$

- σ_i is the elasticity of substitution across inputs:

$$\sigma_i < 1 : \text{complements}, \qquad \sigma_i > 1 : \text{substitutes}, \qquad \sigma_i \to 1 : \text{Cobb–Douglas}.$$

- Downside idiosyncratic productivity shock:

$$u_{j,t} \equiv -\Delta z_{j,t} \geq 0, \qquad Z_{j,t} = \exp(z_{j,t}).$$

Equilibrium Shares and Network Multiplier

- Relative log prices solve the CES unit-cost recursion:

$$p_{i,t} = -z_{i,t} + \frac{\alpha_i}{1-\sigma_i} \log \left(\sum_{j=1}^{N} \omega_{ij,t} \exp\{(1-\sigma_i) p_{j,t}\} \right).$$

- Endogenous expenditure share of customer firm i on supplier good j:

$$s_{ij,t} = \frac{\omega_{ij,t} \exp\{(1-\sigma_i) p_{j,t}\}}{\sum_{k=1}^{N} \omega_{ik,t} \exp\{(1-\sigma_i) p_{k,t}\}}.$$

- Local price-pass-through matrix and high-order network multiplier:

$$B_{ij,t} = \alpha_i s_{ij,t}, \qquad \mathcal{L}_t = [\ell_{ij,t}] = (I - B_t)^{-1} = \sum_{m=0}^{\infty} B_t^m.$$

- $\mathcal{L}_t$ is an equilibrium-share multiplier.

From Network to CDS Spreads

- **Network → fundamentals**

$$E_{i,t} = \bar{E}_{i,t} - \kappa^{E}_{i,t} X^{L}_{i,t}, \qquad X^{L}_{i,t} = \sum_{j=1}^{N} \ell_{ij,t} u_{j,t},$$

$$\sigma_{E,i,t} = \left(\bar{\sigma}^{2}_{E,i,t} + \kappa^{\sigma}_{i,t} X^{V}_{i,t}\right)^{1/2}, \qquad X^{V}_{i,t} = \sum_{j=1}^{N} \ell^{2}_{ij,t} \sigma^{2}_{u,j,t}.$$

- **Fundamentals → distance to default**

$$DD_{i,t} = \frac{\ln(V_{i,t}/F_i) + \left(r - \frac{1}{2}\sigma^{2}_{V,i,t}\right)(T-t)}{\sigma_{V,i,t}\sqrt{T-t}},$$

$$V_{i,t} \approx E_{i,t} + F_i, \qquad \sigma_{V,i,t} \approx \frac{E_{i,t}}{V_{i,t}} \sigma_{E,i,t}.$$

- **Distance to default → CDS spreads**

$$\mathbb{P}^{\mathbb{Q}}_{i,t}(\text{default by } T) \approx \Phi(-DD_{i,t}), \quad s^{CDS}_{i,t} = \frac{1-R}{T-t}\Phi(-DD_{i,t}).$$

Implication 1: Full Network Prices CDS Nonlinearly

- For each firm i, distance to default is a nonlinear function of the full network multiplier:

$$DD_{i,t} = \mathcal{D}_i \left(\mathcal{L}_t; \bar{E}_{i,t}, \bar{\sigma}_{E,i,t}, F_i, r, T - t \right).$$

- Hence CDS spreads satisfy

$$s_{i,t}^{CDS} = \frac{1 - R}{T - t} \Phi \left[-\mathcal{D}_i \left(\mathcal{L}_t; \bar{E}_{i,t}, \bar{\sigma}_{E,i,t}, F_i, r, T - t \right) \right].$$

- Orthogonal negative idiosyncratic shocks propagate through both direct and indirect production paths.

- Therefore, the full network structure becomes the state variable.

Implication 2: Hub-and-Spoke Fragility

- Central supplier c: for customer firms $i \in \mathcal{I}$,

$$s_{ic,t} \geq 1 - \varepsilon, \qquad s_{cc,t} \geq 1 - \varepsilon.$$

- The first condition makes c a dominant supplier to many firms; the second creates a strong self-input feedback loop.

- For a finite downside shock $u_{c,t} > 0$, there exists a degree of centrality such that

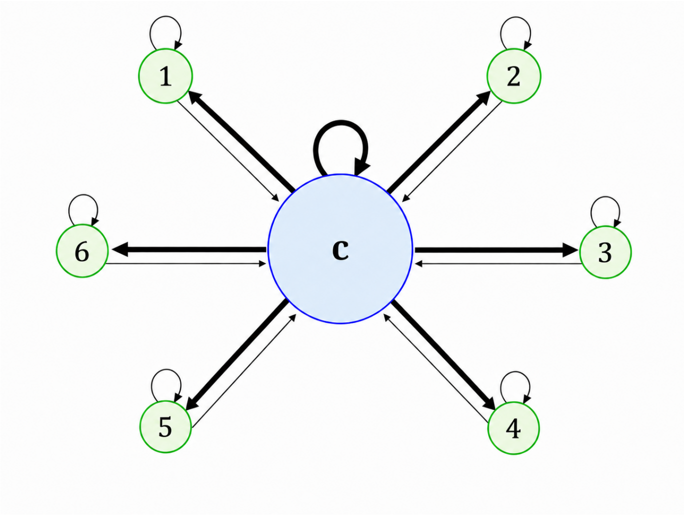
$$DD_{c,t}(u_{c,t}) > \bar{d}_c, \qquad \mathbb{P}^{\mathbb{Q}}_{i,t}(\text{default by } T \mid u_{c,t}) \geq q.$$

- In the bottleneck limit,

$$\lim_{\alpha_c(1-\varepsilon)\uparrow 1} \mathbb{P}^{\mathbb{Q}}_{i,t}(\text{default by } T \mid u_{c,t}) = 1,$$

even though the supplier shock is finite and the supplier remains solvent.

Implication 2: Hub-and-Spoke Fragility



Implication 3: Elasticity and Tail Propagation

- Consider an infinitely large negative productivity shock to supplier firm c:

$$z_t = -xe_c, \qquad x \to \infty.$$

- Prices admit tail exposure coefficients:

$$p_{i,t}(-xe_c) = \mu_i^c + \phi_i^c x + o(x).$$

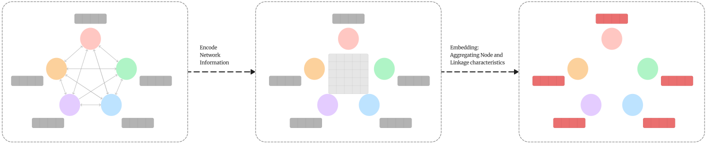
- Tail exposures satisfy

$$\phi_i^c = \mathbf{1}\{i = c\} + \alpha_i \begin{cases} \max_{j\in\mathcal{S}_i} \phi_j^c, & \sigma_i < 1, \\ \sum_{j\in\mathcal{S}_i} \omega_{ij}\phi_j^c, & \sigma_i = 1, \\ \min_{j\in\mathcal{S}_i} \phi_j^c, & \sigma_i > 1. \end{cases}$$

- If $\phi_i^c > 0$, then $DD_{i,t} \to -\infty$ and $\mathbb{P}_{i,t}^{\mathbb{Q}}(\text{default by } T) \to 1$.

Methodology: Graph Neural Network

- GNN algorithm covers two primary schemes:
  - Inter-layer design. $\hat{h}_{\nu}^{(k)} = h_{\nu}^{(k)} \bigoplus_{k \in \mathcal{N}(\nu)} \text{MESSAGE}_{\theta}(h_{\nu}^{(\nu)}, h_{\nu}^{(k)}, e_{\nu,k})$



    The embedding is computed as a weighted average of all neighboring nodes' features, where the weight is determined by edge thickness.

  - Intra-layer design:

    Normalization, activation and aggregation
    - 1-hop neighborhoods
    - 2-hop neighborhoods
    - 3-hop neighborhoods

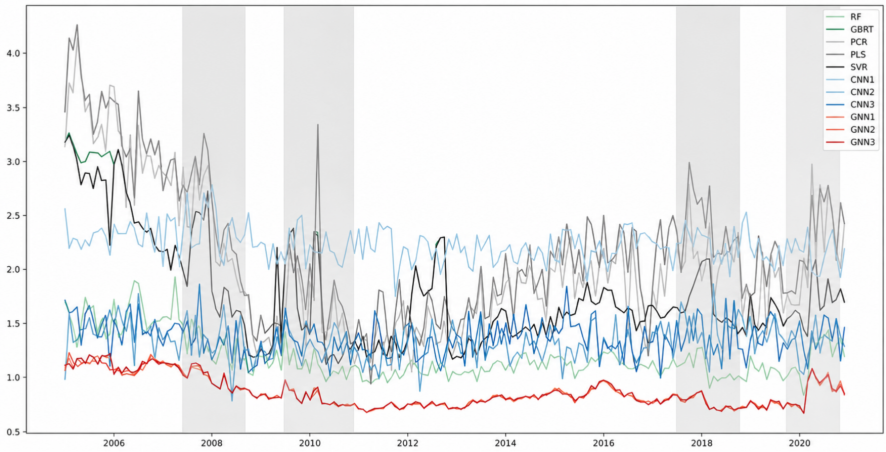
Characteristics

- **Edge Characteristics:**
  - Monthly estimates of pairwise idiosyncratic risk spillovers (Diebold et al 2014), disciplined by yearly input-output table
- **Node Characteristics:**
  - 94 firm-level accounting variables (Gu, Kelly, and Xiu, 2020)
  - 8 macroeconomic variables (Welch and Goyal, 2008), including interactions with firm-level characteristics
- **Target Variable:** 130,176 log CDS spreads (678 firms)
  - Monthly Markit CDS spreads for 5-year tenor contracts, covering all U.S. firms from Jan 2005 to Dec 2020

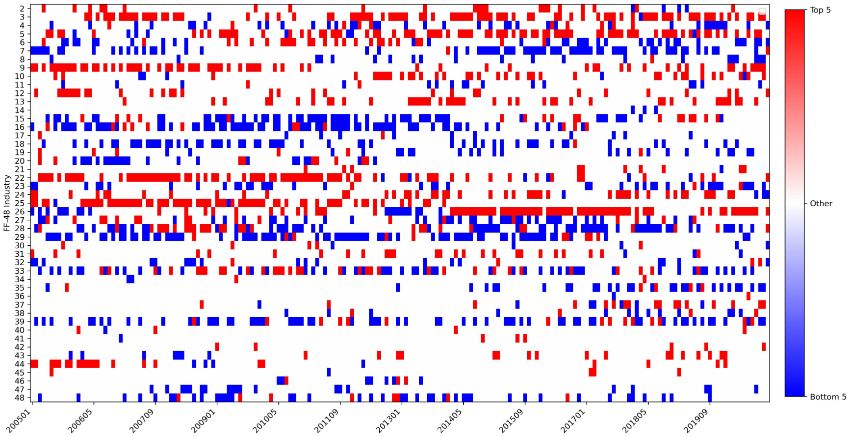
Out-of-Sample Performance

	All	Investment Grade	High Yield	Small	Big
Panel A: Pooled Out-of-Sample RMSE					
$GNN1_{zeroedge}$	2.245	2.308	2.110	2.072	2.405
$GNN2_{zeroedge}$	1.338	1.392	1.221	1.282	1.391
$GNN3_{zeroedge}$	1.378	1.437	1.249	1.337	1.418
GNN1	0.890	0.909	0.850	0.808	0.965
GNN2	0.888	0.904	0.854	0.809	0.960
GNN3	0.893	0.915	0.847	0.808	0.972
Panel B: Network-Attributable Spread Change and Incremental R^2					
NSC	0.218	0.185	0.301	0.266	0.177
Inc_R^2	0.560	0.578	0.511	0.602	0.524

Time Series OOS RMSE



Heterogeneity Across Industries



- Red industries: Stronger network-related CDS spillover effects, including defense, electrical equipment, shipbuilding, and railroad equipment.
- Blue industries: Weaker network-related CDS spillover effects, including textiles, rubber and plastics, mining, and coal, and entertainment.

Conclusion

- We develop a structural framework that links firms' credit risk to the inter-firm production network.
- Network topology is an important determinant of credit spreads.
  - Adding network edge features changes predicted credit spreads by about 21.8% on average and increases explanatory R^2 by 0.56.
  - Network-based counterparty risk becomes especially important during periods of supply-chain disruption, reorganization, or rewiring.
  - Network effects are strongest for firms in central supply-chain positions and firms with low input substitutability.